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Faculty of Textile Engineering

STRUCTURE AND GEOMETRY OF SINGLE AND TWO LAYER STITCHED WOVEN FABRICS

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SUMMARY OF THE THESIS

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Disertační práce je podávána v oboru: Textile Technics and Materials Engineering.

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The dissertation is available at the Dean's Office FT TUL.

Liberec 2019

Abstract

There are different ways of making fabrics but the most common method of producing woven fabric is by interlaced yarns. The woven fabric geometry and structure have significant effects on their behavior. The woven structures provide a combination of strength with flexibility. Woven fabrics are key reinforcements which offer ease of handling, moldability, and improved in plane properties. Most of the composites are made by stacking layers of woven performs over each other which can cause the delamination failure in composite materials. This problem has been tackled by using multilayer woven perform as reinforcement, instead of single layer woven fabrics. The structure and properties of a woven fabric are dependent upon the constructional parameters such as thread density, yarn fineness, crimp, weave etc. As we know, woven fabrics are not capable of description in mathematical forms based on their geometry because these are not regular structures; but many researchers believe that we can idealize the general characters of the materials into simple geometrical forms and physical parameters to arrive at mathematical deductions. It is always assumed that the variation of the fabric structure is insignificant in the analysis. The models given by these researchers can describe the internal geometry of woven fabric by describing some part of the binding wave. But we need a model that can describe binding wave in whole repeat and the validation is good from left or right side. We need to obtain not only geometry of binding wave but also spectral characterization for analyzing individual components, which can react on deformation of the shape of binding wave.

In this study, an attempt is made to create a theoretical model on the geometry of plain single and two layer woven structures and verify them with experimental results. The first part of the work deals with the model development and the second part reports on model validation. In the first part, the basic description of the geometry of woven fabric has been described. Many attempts have been made by different researchers to find a suitable model for describing the binding cell. They have worked mathematically to express the shape of the binding wave in a given thread crossing in a woven fabric in a steady state. The geometric models have been studied to find out their limitations as well. After a comprehensive study, the geometry of binding cell in plain weave for single and two layer stitched woven fabrics have been presented for theoretical evaluation by Fourier series. This study shows some interesting mathematical relationships between constructional parameters of single and two layer stitched woven fabrics, so as to enable the fabric designers and researchers to have a clear understanding of the engineering aspects of single and two layer woven fabrics.

In the second part of the work, the theoretical model for the description of mutual interlacing of threads, in multifilament woven fabric structure using Fourier series, derived from plain woven structure has been validated with experimental results. The internal geometry of the woven fabrics and the deformation of the shape of the binding wave in the single and two layer stitched woven structures has been evaluated by the cross-sectional image analysis method. The approximation using the linear function *f(x)* in Fourier series along longitudinal and transverse cross-section has been performed for single layer and two layer stitched woven fabrics cross-section, which fits well to the experimental binding wave. The spectral characteristics of binding waves obtained by Fourier series (theoretical) has been compared with the experimental values, which are very close to each other in longitudinal and transverse cross-section. By evaluating the geometrical parameters of yarn in the real cross-section of a woven fabric, it is possible to compare it with the theoretical shape of a binding wave by analyzing its individual coordinates. The approximation of the whole binding repeat by a partial sum of FS with straight lines description of central line of the binding wave has also been performed for different repeat sizes and compared with each other to analyze the difference in spectrum.

Keywords: Weaving, fabric structure, geometry, stitched woven fabrics, Fourier Series, multifilament, reinforcement fabrics.

Anotace

Existují různé způsoby výroby textilií. Jednou z možností výroby je výroba na základě technologie tkaní. Kde tkanina vzniká vzájemným provázáním osnovních a útkových nití. Geometrie a struktura tkanin má významný vliv na její chování. Tkaniny jako jeden ze tří plošných útvarů jsou klíčové výztuhy, které nabízejí snadnou manipulaci, tvárnost a zlepšují rovinné vlastnosti. Většina kompozitů je vyrobena vrstvením z tkaných materiálů, kde může nastat separace jednotlivých kompozitních vrstev výztuže. Tento problém může být řešen pomocí použitím vícevrstvé tkané výztuže spojkové, místo jednoduché tkaniny. Struktura a vlastnosti tkanin jsou závislé na konstrukčních parametrech, jako je jemnost nití, dostava (osnovy a útku), vazba, setkání atd. Jak je známo, tkaniny možné popsat pomocí matematických forem založených na jejich geometrii. Lze idealizovat obecné charakteristiky materiálů do jednoduchých geometrických tvarů a fyzikálních parametrů, k vytvoření matematické formulace. Modely mohou popisovat vnitřní geometrii tkanin popisem některé části vazné vlny. Avšak my potřebujeme model, který dokáže popsat vaznou vlnu jako celek – celou střídu vazby.

V této studii se usiluje o vytvoření teoretického modelu geometrie jednoduché a dvouvrstvé tkané struktury a jejich ověření s experimentálními výsledky. První část práce se zabývá vývojem modelu a druhou částí je zpráva o ověření tohoto modelu. V první části, je líčen základní popis geometrie tkanin. Řada výzkumníků učinila mnoho pokusů najít vhodný model pro popis vazebné buňky. Byly vytvořené matematické modely pro vyjádření tvaru vazebné vlny v příčném řezu plátnového provázání v ustáleném stavu. Tyto geometrické modely byly také studovány z hlediska nalezení jejich limitních hodnot provázání. Po obsáhlých studiích byla geometrie vazné buňky (pro jednoduché a dvouvrstvé tkaniny spojkové) prezentována jako teoretické hodnocení využívající Fourierových řad. Tato studie ukazuje některé zajímavé matematické vztahy mezi konstrukčními parametry jednoduché a dvouvrstvé tkaniny spojkové.

Ve druhé části této práce, byl ověřen teoretický model pro popis vzájemného provázání nití ve struktuře jednoduchých tkanin s plátnovou vazbou s využitím Fourierových řad. Teoretické modely byly porovnaný s experimentálními hodnotami získanými z reálné vazné vlny pomocí obrazové analýzy. Vnitřní geometrie tkaniny a deformace nití ve struktuře tkaniny s jednou a dvěma vrstvami byly hodnoceny metodou analýzy obrazu. Pro jednoduchou a dvouvrstvou tkaninu spojkovou v podélném a příčném řezu byla provedena analýza využitím Fourierových řad, kde vstupní funkce k vyjádření popisu byla použita lineární funkce $f(x)$. Spektrální charakteristika, včetně popisu střednice vazné vlny získaných pomocí Fourierovy řady (teoretické) byla porovnána s experimentálními hodnotami, které jsou v podélném pohledu a příčném průřezu velmi blízké. Hodnocením geometrických parametrů osnovních a útkových nití v reálném průřezu tkaniny je možné porovnávat s teoretickým tvarem vazné vlny pomocí analýzy jejích jednotlivých souřadnic. V rámci práce bylo provedeno hodnocení a porovnání provázání a struktury tkaniny pro různé opakované velikosti střídy dvouvrstvé spojkové tkaniny. Jak je patrné z výsledného hodnocení, poloha a velikost spojky přímo určuje tvar spektrální charakteristiky vycházející z daného rozvoje Fourierovy řady použitého pro konkrétní popis tvaru vazné vlny spojkové dvouvrstvé tkaniny.

Klíčová slova: Tkaní, struktura tkanin, geometrie, tkaniny, Fourierovy řady, multifilament, výztuže, vazba.

Summary

1 Introduction

In recent years, the woven fabrics have gained much attention because of their superior properties over conventional materials used in engineering structures. The woven fabric geometry and structure have significant effects on their behavior. For example, for the multilayer structures, a good reinforcement material should be chosen as a good reinforcement material ensures better properties of a final product. Whereas, to access the better properties of reinforcement material, it is very important to understand the internal geometry of the woven fabric [1]–[3].

The study of fabrics mechanics requires special attention due to the large deflection effects and the nonlinearity of the textile structures deformation phenomena. As we know, woven fabrics are not capable of description in mathematical forms based on their geometry because these are not regular structures; but many researchers believe that we can idealize the general characters of the materials into simple geometrical forms and physical parameters to arrive at mathematical deductions. Researchers have put forward many different forms of fabric geometry to represent the configuration of threads in woven fabrics. It is very complex to build up a direct mathematical relationship to predict the structural properties of the woven fabric [4]. Moreover, it is not possible to rely only on theoretical models, however it can be combined with empirical findings as well. In this regard, it is necessary to find out the best possible approach, which may use a special method of analyzing the cross-section of woven structures to evaluate the fabric geometry in a better way and to correlate it with predicted theoretical findings.

The main aim of this work is the analysis of mutual interlacing of threads in multifilament single layer and two layer stitched woven fabric structure by using the proposed methodology which is based on Fourier series.

2 Purpose and aim of the thesis

The purpose of the work is the description and expression of geometry of woven fabric structure - two layer stitched woven fabrics with plain weave in the cross-section. Evaluation and analyzation of cross-sectional image of single layer and two layer stitched woven fabric structures with plain weave, which can be used as reinforcement fabrics. The main aim is the creation of model as well as the development of methodology to analyze the shape of binding wave as well as yarn deformation in single layer and two layer stitched woven fabrics with plain weave and to validate it with theoretical models. The work has been divided into the following parts.

1. Understanding the behavior of fiber and yarn by analyzing its physical properties like fineness, diameter, twist per meter, tenacity and elongation etc.
2. To prepare the single and two layer stitched woven structures with different material, weft settings and stitching (connection) points on a sample weaving loom.
3. Study of basic geometric models and analyzing their limitations. An improved theoretical model for the description of geometry of cross-section of woven multifilament fabric structure – single and two layer stitched woven fabrics with plain weave will be presented.
4. The evaluation of the internal geometry of the woven fabrics and analysis of deformation in the single and two layer stitched woven structures by the cross-sectional image analysis method.
5. Performing the Fourier analyses by the mathematical modeling of geometry of binding wave in woven fabric structure using Fourier series. Mathematical modelling creates information about shape – geometry of binding wave and characteristic of weave and interlacing - the spectrum for single layer and two layer stitched woven fabrics with plain weave. The approximation of two layer stitched woven fabrics with different repeat size will be performed and their spectrum will be analyzed as well.
6. The validation of experimental values by the proposed theoretical model which is going to be proposed for the evaluation of single layer and two layer stitched woven structures.

3 Overview of the current state of the problem

Textile structures are recognized for their exclusive combination of light weight, flexibility and their capability to offer a combination of strength and toughness [5]. The textile structures are inhomogeneous, anisotropic, porous materials with distinct viscoelastic properties [6]. The strongest growth potential for advanced structures is in high-performance applications such as aerospace, maritime, transport, or construction industries. The properties demanded by these applications are met by continuous fiber reinforced composites. Woven fabrics are key reinforcements which offer ease of handling, moldability, and improved in plane properties [7], [8].

To understand the internal geometry of woven fabric, which refers to the spatial orientation of yarns in the structure of a fabric, many studies have been performed in the past [9], [10].

Pierce’s, Kemp’s, Olofsson’s and Hearl’s model are known as the most used and best-known models [11]–[14]. Moreover, there are some investigations in which these mentioned models has been compared and evaluated [15], [16]. The principles on which all these models are based remain unaltered. It is always assumed in these models that the geometric shape is constant for each model of the unit cell or it can be said the variation of the fabric structure was considered insignificant in the analysis.

Fourier transform has been applied by Jaume et al. on woven fabric structures by image analysis, which is a non-destructive and non-contact testing technique to obtain the fabric structure or the pattern of weaving [17]. Similarly, Bohumila and Stanislav applied discrete Fourier transform (DFT) on the determination of yarn waviness for eight-layer carbon composites [18]. Whereas it has been described by Brigita Sirkova that by using the sum of Fourier series, the spectral characteristic of the approximated course can be obtained. The spectral characteristic consists of amplitude and phase characteristics of individual wavelengths [19]. Kawabata et al. developed a 3D sawtooth geometry, which allows the implementation of biaxial response in a mechanistic way. The warp and weft axes are assumed to be straight lines for simplifications. The current work focuses on a macroscopic length scale geometrical model for woven fabrics using the mesoscopic sawtooth geometry developed by Kawabata et al. [20].

The analyzation of the shape of the binding wave, yarn deformation and mutual interlacing in in the whole weave repeat of plain woven fabrics has not been possible by the described models earlier so in this study by using Fourier series a model as well as a methodology will be developed to analyze the shape of the binding wave in the whole weave repeat of multifilament single layer and two layer stitched woven fabric structure.

4 Theoretical modelling of woven fabric geometry structure

Woven fabrics are composed of two distinct set of yarns, which are interlaced at right angles to each other. The longitudinal yarns are called the warp and lateral yarns are the weft or filling [21]. The pattern of interlacing and weave diagram of these two set of yarns for plain weave has been shown in Figure 1. Many attempts have been done in the past to find a suitable model describing the binding cell, i.e. to express mathematically the shape of the binding wave in each thread crossing in woven fabric in the steady state. Pierce model, Ollofsson model, hyperbolic model, parabolic shapes are registered as the most used and best-known models [22].

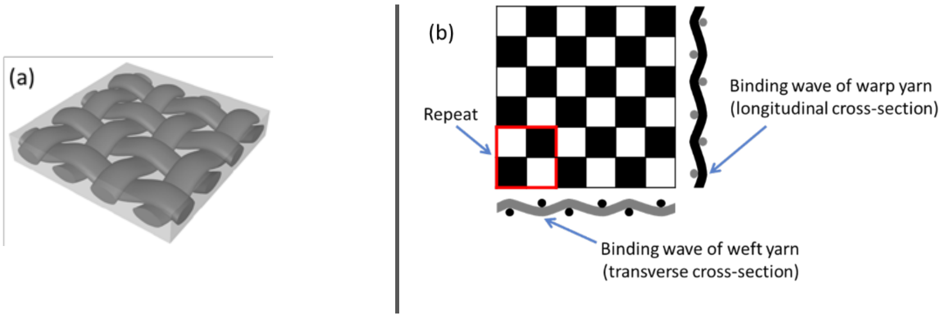


Figure 1. Plain woven fabric (a) Interlacing of warp and weft yarn (b) Weave diagram and cross-sectional view

A mathematical model that permits the characterization of fabric structure and surface profile is to be developed. For simplification, here we investigate only a model for woven fabrics with basic weave patterns. One factor that is common to all weave designs is that of the repeating pattern or the unit cell. The unit cell represents the smallest repeat unit of the weave architecture and describes the whole reinforcing fabric, so it is necessary to understand it [23]. There are some assumptions taken for yarn as it is monofilament, smooth, circular in cross-section, crimp is ideal and both warp and weft are perpendicular to each other and evenly distributed, without any distortion and having same diameters [24].

4.1 Geometry of binding cell in plain weave for single and two layer woven fabric and parameters description of woven fabrics

The woven fabric is treated as an assembly of unit cells and the unit cell is the smallest repeating pattern in the structure. The plain fabric is created by the mutual interlacing of two set of threads. The manner of the mutual interlacing of threads defines the final structure of the fabric. The shape of the binding (crimp) wave and basic geometry of the binding cell changes according to the dimension and number of threads in the weave repeat [25]. The geometry of the binding cell is characterized by the following parameters and dimensions:

T_1 and T_2 = the yarn count for warp and weft yarn
D_1 and D_2 = setting of warp and weft threads
d_1, d_2 and d_s = yarn diameters for warp and weft and their mean diameter
A and B = the distances of warp and weft threads
e_1 and e_2 = relative waviness of warp and weft yarn in single layer woven fabrics
h_1 and h_2 = the heights of warp and weft binding waves in single layer woven fabrics
n_1 and n_2 = number of ends and picks in weave repeat

The geometry of the binding point in the plain weaves, for the mathematical model used later, is based on the Brierley theory of the tight weave (Figure 2) [26]. A woven fabric in which warp and weft yarns do not have mobility within the structure as they are in intimate contact with each other are called jammed structures. In such structures the warp and weft yarns will have minimum thread spacing and their geometry is called limit geometry. These are closely woven fabrics and find applications in wind-proof, water-proof and bullet-proof requirements [27]. Whereas the structures other than limit are called looser structures, in which warp and weft yarn have some thread spacing and mobility within the structures. The two layer stitched woven fabric is composed of two layers of simple woven fabric and these layers are joined together by the interlaced or binder yarn. The limit, semi-loose and looser geometry of two layer stitched woven fabrics with plain weave have been derived from simple plain weave in single layer woven fabrics and it can be observed in Figure 3, Figure 4 and Figure 5 respectively.

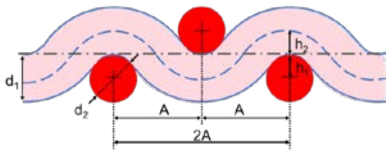


Figure 2. Limit geometry of single layer woven fabrics with plain weave

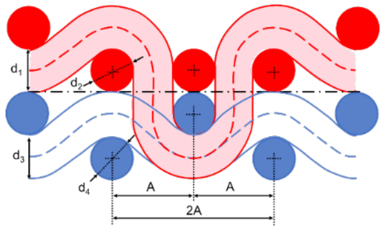


Figure 3. Limit geometry of two layer stitched woven fabrics with plain weave

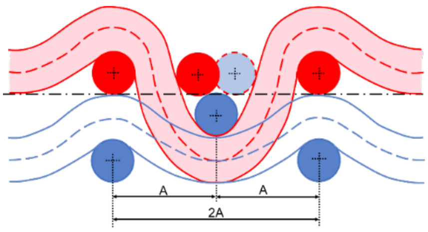


Figure 4. Semi-loose geometry of two layer stitched woven fabrics with plain weave (geometry between limit and loose)

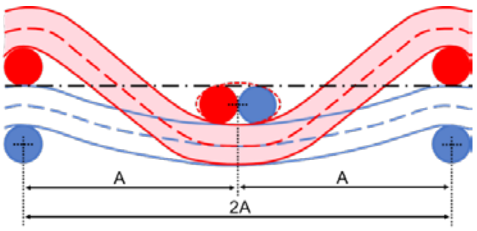


Figure 5. Looser geometry of two layer stitched woven fabrics with plain weave

The geometry of two layer stitched woven fabrics with plain weave can be divided into stitching and non-stitching section as shown in Figure 6. While the geometry of the binding cell for other than plain weaves can be derived from the plain weave as well. The looser interlacing of the fabric depends on the type of material being used and on the type of machine.

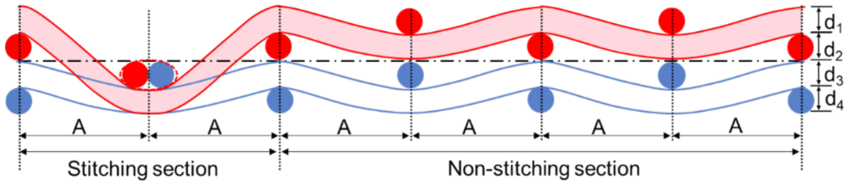


Figure 6. Geometry of two layer stitched woven fabrics with plain weave (stitched and non-stitched section)

The parameters which are necessary for the calculation of the binding waves are given below.

$$A = \frac{1}{D_1} \quad , \; B = \frac{1}{D_2} \tag{1}$$

$$d_s = \frac{d_1 + d_2}{2} \tag{2}$$

$$d_{effective} = d_s * g \tag{3}$$

Where ‘g’ is the constant of deformation obtained from experimental values of yarn cross-sectional diameter. The difference between the theoretical yarn diameter and the experimental diameter in the woven fabric has been obtained which gives us the value of constant ‘g’. It can be used to obtain the effective diameter and as output for modelling of this type of structure and material. The parameter (h_1 and h_2) height of binding waves can be determined based on:

a. Experimental methods - from transverse and longitudinal cross-section of woven fabric by using image analyse, the real heights of the warp and weft crimp waves from fabric centre line can be obtained.
b. Theoretical methods – it is necessary to know effective diameter of threads and rate of warp and weft waviness e_1 and e_2, which can be used in equation (4) and equation (5) for determination of heights. The rate of thread waviness e_1 and e_2, can be estimated on the basis of individual phases of interlacing from Novikov work [28].

$$h_1 = e_1 . d_{effective} \, , \qquad h_2 = e_2 \, . \, d_{effective} \tag{4}$$

$$e_1 + e_2 = 1 \tag{5}$$

The distance between warp and weft yarn axis and $e_1 = e_2 = 0.5$ is given by equation (6). This equivalency is valid every time, independently to a theoretical model used.

$$h_1 + h_2 = (d_1 + d_2)/2 \tag{6}$$

4.2 Mathematical model for the description of binding wave by using Fourier series (FS)

The description of the shape of binding waves can be provided in the fabric, in the longitudinal cross-section (the shape of the binding wave of the warp thread) and in the transverse cross-section (the shape of the binding wave of the weft thread), to define the mutual position of the warp threads towards weft threads.

Due to spatial threads distribution in the cross-section of a cloth, the shape of the binding waves obtains the form which is near to the harmonic sinus course. That leads to the idea to approximate the binding wave by a sum of Fourier series (FS). For the weave of the fabric, as it is characteristic that the pattern of binding is repeated regularly (periodically) across the whole fabric width, and that it is continuous. The Fourier approximation respects this periodicity and shape of the binding wave, in the contrary to the above-mentioned models of single threads crossing. In our case of the periodically repeated pattern of thread waves, it means to substitute the binding wave by a system of sine curves with increasing frequencies (decreasing wavelengths), with different amplitudes and phase shifts. Apart from the approximated course, we also obtain the spectral characteristic of the course, by approximations using the sum of Fourier series. Spectral characteristic consists of amplitude and phase characteristics of individual wavelengths. The wavelengths are the whole fractions of the basic wavelengths of the pattern on the interval of the binding repeat (0, binding repeat).

For the creation of spectral characteristics of the repeat of binding in the longitudinal and transverse cross-sections, it is necessary first to describe the spectral characteristics of individual binding waves in the binding repeat in both the longitudinal as well as transverse cross-section. It has been said, that in the basic weaves, the interlacing of threads is identical in the longitudinal as well as in the transverse cross-section (using identical parameters of threads). This is not true for derived higher weaves or special weaves. The shapes of the binding waves in the derived weaves are different in the longitudinal and transverse section. Resulting from individual sections, spectral characteristics of the repeat of the binding in the derived weaves will be different for the longitudinal and for the transverse cross-sections. The difference of individual spectral characteristics depends on the number of threads (binding waves) in the binding repeat and on their mutual interlacing. The shape of the binding wave or its course can be possibly obtained by two methods:

4.2.1 FS approximation of binding wave (theoretical general description of model)

There is a tendency to avoid labored and tardy procedure of the creation of the cross-sections experimentally. The values of the course of one binding wave will be obtained by substitution of the wave shape by a well-known analytic function $f(x)$ (linear, circular, parabolic, hyperbolic, etc.), or created by a sum of functions defined on the specified interval 'T'. The interval is given by the width of the repeat of binding. For a function $f(x)$, periodic on an interval $[0, T]$, the Fourier series of a function $f(x)$, is given by.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n.2.\pi.x}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n.2.\pi.x}{T}\right) \tag{7}$$

Where the coefficients are,

$$a_0 = \frac{2}{T}\int_0^T f(x)\,dx \tag{8}$$

$$a_n = \frac{2}{T}\int_0^T f(x)\,.\cos\left(\frac{n.2.\pi.x}{T}\right)dx \qquad (n = 0,1,2,\dots) \tag{9}$$

$$b_n = \frac{2}{T}\int_0^T f(x)\,.\sin\left(\frac{n.2.\pi.x}{T}\right)dx \qquad (n = 1,2,3,\dots) \tag{10}$$

During the modelling and searching for certain dependencies, it is necessary to consider the equilibrium between the efficiency of the used model and its accuracy to the expressed parameter. This model has been extended for single layer and two layer stitched woven fabrics and explained further.

A) Modelling of binding wave of single layer plain woven fabric cross-section

Mathematical expression of geometry of binding wave using Fourier series - construction of structure of woven fabric of single layer of plain woven fabric geometry

Modelling of central line of threads in cross-section in woven fabric is based on Fourier series. For mathematical definition of binding wave, in this case as an input function f(x) in Fourier series, it is possible to use different mathematical shapes like linear, circular arc, parabolic, hyperbolic, sine or rectangular description [19]. Based on literary research [29]–[31] for mathematical modelling of binding wave using the Fourier series, it is sufficient that the simplest description of the central line of the binding wave is given by the linear description by means of two straight lines as shown in Figure 7 for single layer woven fabric. This description is applicable in every interlacing of basic weaves as well as of higher derived weaves. It allows the evaluation of the warp and weft threads in the interlacing. In the case of single layer plain weave, it represents the weaving with the simplest interlacing, therefore, only two different interlacing threads appear in the binding repeat along the longitudinal and transverse cross-sections.

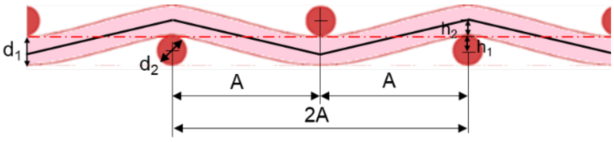


Figure 7. Geometry and graphical illustration of linear description of central line of thread in cross-section of single layer woven fabrics with plain weave

For the approximation of a single layer woven fabric by a partial sum of FS with straight lines description of central line of the binding wave, the period of the periodic function has been taken as $P = T = 2A$ in single layer woven fabrics and parameters mentioned below have been taken.

$$h_2 = 59\ \mu m, \qquad A = 1220 \mu m, \qquad d_s = 280\ \mu m, \qquad g = 0.6$$

The equations of the linear functions were used in the Fourier equations to find the coefficients a_0, a_n and b_n. The final equations are;

$$a_0 = \frac{2}{T}\left[\int_0^A (k.x + h_2)\,dx + \int_A^{2A} (-k.(x-A) - h_2)\,dx\right] \tag{11}$$

$$a_n = \frac{2}{T}\left[\int_0^A (k.x + h_2)\cos\left(\frac{n.2.\pi}{T}x\right)dx + \int_A^{2A} (-k.(x-A) - h_2)\cos\left(\frac{n.2.\pi}{T}x\right)dx\right] \tag{12}$$

$$b_n = \frac{2}{T}\left[\int_0^A (k.x + h_2)\sin\left(\frac{n.2.\pi}{T}x\right)dx + \int_A^{2A} (-k.(x-A) - h_2)\sin\left(\frac{n.2.\pi}{T}x\right)dx\right] \tag{13}$$

Where k is the slope of the linear function in single layer plain woven fabric. The final approximation function is,

$$F_\alpha(x) = \frac{a_0}{2} + \sum_{n=1}^{\alpha} a_n \cos\left(\frac{n.2.\pi}{T}x\right) + \sum_{n=1}^{\alpha} b_n \sin\left(\frac{n.2.\pi}{T}x\right) \qquad (\alpha = 1,2,3,\dots) \tag{14}$$

By applying Fourier approximations, it changes the shape and gives us the shape which is comparable to the real shape of binding wave, which will be explained later in experimental data validation.

B) Modelling of influence of plain weave repeat in non-stitched part of binding wave of two-layer stitched woven cross-section

a. Mathematical expression and description of binding wave in two-layer stitched cross-section of woven fabric - construction of structure of woven fabric with minimum time plain weave repeat in non-stitching section

The geometry of two layer stitched woven fabrics with plain weave can be divided into stitching and non-stitching section as shown by upper binding wave in Figure 8. For regular repeat of the interlacing the minimum number of plain weave repeat in non-stitching section is one-time as given in figure. The linear description of the central line of the binding wave in two layer stitched woven fabric has also been illustrated in Figure 8. For the approximation of a two layer woven fabric by a partial sum of FS with straight lines description of central line of the binding wave, the period of the periodic function has been taken as $P = T = 4A$. Some additional parameters have been described for the geometry of the binding cell of two layer stitched woven fabric as under.

h_{f^1} and h_{f^2} = the height of first warp and weft binding waves in two layer woven fabrics
h_{s^1} and h_{s^2} = the height of second warp and weft binding waves in two layer woven fabrics
e_{f^1} and e_{f^2} = relative waviness of first warp and weft yarn in two layer woven fabrics
e_{s^1} and e_{s^2} = relative waviness of second warp and weft yarn in two layer woven fabrics
h'_{f^2} = the height of first warp binding waves in two layer woven fabric in non-stitching section ($h_{f^2} \neq h'_{f^2}$)

$$h'_{f^2} = h_{f^2} - (h_1 + h_2) \tag{15}$$

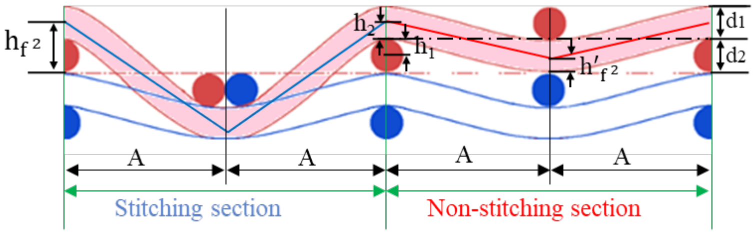


Figure 8. Geometry and graphical illustration of linear description of central line of thread in cross-section of two layer stitched woven fabrics with plain weave

The equations of the linear functions were used in the Fourier equations to find the coefficients a_0, a_n and b_n. There are four linear equations in these coefficients, the first two are for stitching section and the last two are for non-stitching section. The final equations are given by:

$$a_0 = \frac{2}{T}\left[\left(\int_0^A (k_1.x + h_{f^2})\,dx + \int_A^{2A} (-k_1.(x-A) - h_{f^2})\,dx\right) + \left(\int_{2A}^{3A} (k_2.(x-2A) + h_{f^2})\,dx + \int_{3A}^{4A} (-k_2.(x-3A) + h'_{f^2})\,dx\right)\right] \tag{16}$$

$$
\begin{aligned}
a_n = \frac{2}{T}\Bigg[\Bigg(&\int_0^A (k_1.x\ +\ h_{f^2})\cos\left(\frac{n.2.\pi}{T}x\right)dx \\
&+\int_A^{2A} (-k_1.(x-A)-\ h_{f^2})\cos\left(\frac{n.2.\pi}{T}x\right)dx\Bigg) \\
&+\Bigg(\int_{2A}^{3A} (k_2.(x-2A)\ +\ h_{f^2})\cos\left(\frac{n.2.\pi}{T}x\right)dx \\
&+\int_{3A}^{4A} (-k_2.(x-3A)+\ h'_{f^2})\cos\left(\frac{n.2.\pi}{T}x\right)dx\ \Bigg)\Bigg]
\end{aligned}
\qquad (17)
$$

$$
\begin{aligned}
b_n = \frac{2}{T}\Bigg[\Bigg(&\int_0^A (k_1.x\ +\ h_{f^2})\sin\left(\frac{n.2.\pi}{T}x\right)dx \\
&+\int_A^{2A} (-k_1.(x-A)-\ h_{f^2})\sin\left(\frac{n.2.\pi}{T}x\right)dx\Bigg) \\
&+\Bigg(\int_{2A}^{3A} (k_2.(x-2A)\ +\ h_{f^2})\sin\left(\frac{n.2.\pi}{T}x\right)dx \\
&+\int_{3A}^{4A} (-k_2.(x-3A)+\ h'_{f^2})\sin\left(\frac{n.2.\pi}{T}x\right)dx\ \Bigg)\Bigg]
\end{aligned}
\qquad (18)
$$

Where k_1 and k_2 are the slopes of the linear functions in two layer stitched woven fabrics in stitching and non-stitching sections respectively. The final approximation function is,

$$
F_\alpha(x) = \frac{a_0}{2} + \sum_{n=1}^{\alpha} a_n \cos\left(\frac{n.2.\pi}{T}x\right) + \sum_{n=1}^{\alpha} b_n \sin\left(\frac{n.2.\pi}{T}x\right) \qquad (\alpha = 1,2,3,\ldots) \qquad (19)
$$

b. Mathematical expression and description of binding wave in two-layer stitched cross-section of woven fabric - Construction of structure of woven fabric with maximum (j) time plain weave in non-stitching section

If we have more number of plain weave repeats in non-stitching section, then we can illustrate the geometric description accordingly and the equation for FS will be different. Suppose we have 'N' number of times of plain weave in non-stitching section (Figure 9). In this case the equation for non-stitching section will be the sum of 'N' number of times of plain repeat.

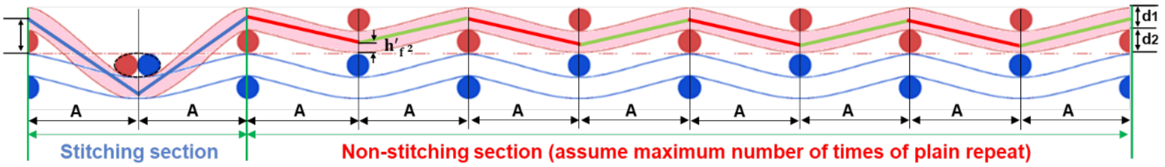


Figure 9. Graphical illustration of linear description of central line of thread in cross-section of two layer stitched woven fabrics with plain weave (assuming it maximum time)

The equations of the linear functions will be used in the Fourier equations to find the coefficients a_0, a_n and b_n. It is explained in the final equations given below:

$$\begin{aligned} a_0 = \frac{2}{T}\Bigg[\Bigg(\int_0^A (k_1.x + h_{f^2})\,dx + \int_A^{2A} (-k_1.(x-A) - h_{f^2})\,dx\Bigg) \\ + \Bigg\{\Bigg(\int_{(j)A}^{(j+1)A} (k_2.(x-j.A) + h_{f^2})\,dx\Bigg) \\ + \int_{(l)A}^{(l+1)A} (-k_2.(x-l.A) + h'_{f^2})\,dx\Bigg\}\Bigg] \end{aligned} \tag{20}$$

Where, $\quad j = 2,4,6,\ldots, number\ of\ repeats\ (only\ even\ numbers)$

and $\quad l = 3,5,7,\ldots, number\ of\ repeats\ (only\ odd\ numbers)$

$$\begin{aligned} a_n = \frac{2}{T}\Bigg[\Bigg(\int_0^A (k_1.x + h_{f^2})\, cos\left(\frac{n.2.\pi}{T}x\right) dx \\ + \int_A^{2A} (-k_1.(x-A) - h_{f^1})\, cos\left(\frac{n.2.\pi}{T}x\right) dx\Bigg) \\ + \Bigg\{\Bigg(\int_{(j)A}^{(j+1)A} (k_2.(x-j.A) + h_{f^2}) cos\left(\frac{n.2.\pi}{T}x\right) dx\Bigg) \\ + \int_{(l)A}^{(l+1)A} (-k_2.(x-l.A) + h'_{f^2}) cos\left(\frac{n.2.\pi}{T}x\right) dx\Bigg\}\Bigg] \end{aligned} \tag{21}$$

$$\begin{aligned} b_n = \frac{2}{T}\Bigg[\Bigg(\int_0^A (k_1.x + h_{f^2})\, sin\left(\frac{n.2.\pi}{T}x\right) dx \\ + \int_A^{2A} (-k_1.(x-A) - h_{f^2})\, sin\left(\frac{n.2.\pi}{T}x\right) dx\Bigg) \\ + \Bigg\{\Bigg(\int_{(j)A}^{(j+1)A} (k_2.(x-j.A) + h_{f^2}) sin\left(\frac{n.2.\pi}{T}x\right) dx\Bigg) \\ + \int_{(l)A}^{(l+1)A} (-k_2.(x-l.A) + h'_{f^2}) sin\left(\frac{n.2.\pi}{T}x\right) dx\Bigg\}\Bigg] \end{aligned} \tag{22}$$

Where k_1 and k_2 are the slopes of the linear functions. The final approximation function is,

$$F_\alpha(x) = \frac{a_0}{2} + \sum_{n=1}^{\alpha} a_n\, cos\left(\frac{n.2.\pi}{T}x\right) + \sum_{n=1}^{\alpha} b_n\, sin\left(\frac{n.2.\pi}{T}x\right) \qquad (\alpha = 1,2,3,\ldots) \tag{23}$$

4.2.2 Mathematical expression and description of real binding wave using Fourier series (experimental analyses of binding wave in cross-section of woven fabric)

It is the approximation of the whole binding wave course obtained experimentally by a partial sum of Fourier series (harmonic synthesis). The series is given by the table of equidistant co-ordinates which will be obtained from the real fabric (real longitudinal and transverse cross-sections) by the visual analysis.

The basic parameters of binding wave in real conditions are detected based on image analysis (using the software NIS elements as shown in Figure 10), which are needed to obtain the coefficient of Fourier series, i.e. a_0, a_n and b_n. The period of the periodic function can be taken as $P = T$, which is the total length of the periodic wave. The coefficients of FS are;

$$a_0 = \frac{2}{m}\,(y_i) \tag{24}$$

$$a_n = \frac{2}{m}\sum_{i=1}^{m}\left[y_i.\,cos\left(\frac{n.2.\pi.i}{m}\right)\right] \tag{25}$$

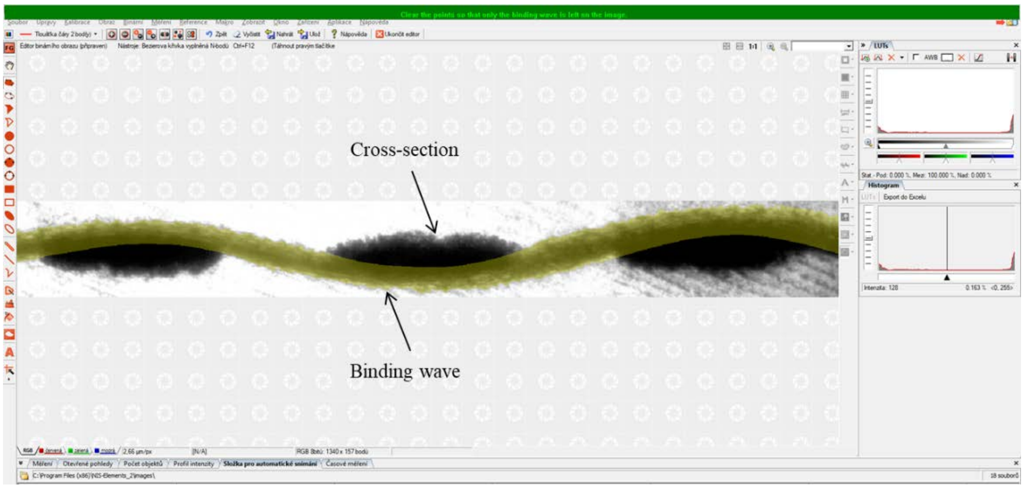
$$b_n = \frac{2}{m}\sum_{i=1}^{m}\left[y_i.\,sin\left(\frac{n.2.\pi.i}{m}\right)\right] \tag{26}$$

Where

m = number of Intervals $(0, T)$

y_i = function value of course at a given point x_i $\quad (x_i = x + ih; \quad h = \frac{T}{m}; \quad i = 0,\ldots,m),$

n = harmonic component.



Clear the points so that only the binding wave is left on the image.

Figure 10. Single layer plain woven fabric with its real cross-section, binding wave (image analysis software NIS element)

The application of Fourier transformation enables to find the wavelength and phase angle of crimp wave as well. For individual components amplitude and phase shift pays the same relationship as in the case of normal equation and are given by;

$$A_n = \sqrt{(a_n)^2 + (b_n)^2} \tag{27}$$

$$\phi_n = atan(\frac{b_n}{a_n}) \tag{28}$$

The final approximation function is;

$$T_\alpha(x) = \frac{a_0}{2} + \left[\sum_{n=1}^{\alpha} A_n\, sin\left(\frac{n.2.\pi}{T}x + \phi_n\right)\right] \qquad (\alpha = 1,2,3,\ldots) \tag{29}$$

5 Used materials, studied methods

5.1 Materials

Basalt 66x2 Tex continuous multifilament plied twisted yarn was used as a raw material. The plied twisted yarn with small number of twist has been used to avoid fibrillation during weaving of multifilament yarn. Sample weaving loom (CCI SL-7900) was used to make the woven structures, keeping same yarn count in warp and weft. All other parameters like weft insertion speed and warp tension was kept the same for all fabric samples. Woven samples were produced with different pick density, stitch distance, weave and number of layers, according to the experimental plan shown in Table 1 [32]. The sample B1-B3 are single layer woven structures with plain weave and B4-B7 are two layer stitched woven structures. The basic fabric structures for single layer and two layer stitched woven fabrics are shown in Figure 11.

Table 1. Construction parameters of woven fabrics

Sample code	**Ends/ cm**	**Picks/ cm**	**Design**	**Sample code**	**Ends/ cm**	**Picks /cm**	**Design**
B1	8.1	6.5	Single layer	**B4**	16.2	18.2	Two layer stitched (S.D.= 0.5 x 0.5)
B2	8.1	8.5	Single layer	**B5**	16.2	18.2	Two layer stitched (S.D.= 1x1)
B3	8.1	10.8	Single layer	**B6**	16.2	18.2	Two layer stitched (S.D.= 1x1.5)
				B7	16.2	18	Two layer stitched (S.D.= 1x2)

*B = Basalt, S.D. = stitch distance (cm)

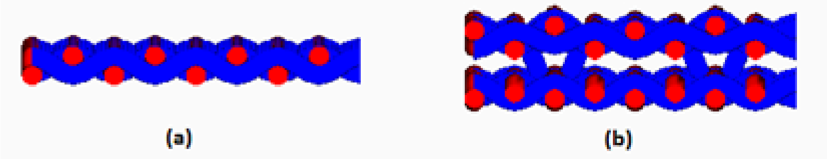


Figure 11. (a) Single layer plain woven fabric, (b) Two layer stitched plain woven fabric

5.2 Methodology

The tensile properties of Basalt yarn were tested using TIRA 2300 instrument in accordance with ASTM D885 [33], while yarn twist per meter was measured by MesdanLAB twist tester machine according to standard procedure ISO 2061:1995 [34]. Similarly, the yarn linear density was measured from a lea of one hundred meters according to standard test procedure ISO 1144:2016 [35]. Yarn diameter was determined by longitudinal view method with the help of instrument in accordance to IS 22-102-01/01[36]. The image analysis of the yarn was performed by software LUCIA.

5.2.1 Cross-sectional image analysis

To measure the geometry of the yarn cross-section in fabric, the fabric samples are impregnated in the epoxy mixture and pre-curing and post-curing was performed. The prepared hard bodies were then cut into pieces of 3 mm thickness in a manner that the cutter was perpendicular to the fabric surface and one group of either warp or weft ends to cut them vertically. Then the samples were fixed into tinny tubs having notch of 4 mm. The epoxy resin was poured into these tubs and upon cooling of epoxy in the tubs, the samples were placed in the oven at 80^oc for 60 minutes (post curing). Later, the slicing was performed by precision saw cutter and the real images has been taken using the projection microscope equipped with digital camera [37]. The test procedure with all the steps involved for cross-sectional image analysis of woven fabric has been shown in Figure 12.

5.2.2 Image processing technique

The cross-sectional image analysis of the fabrics was performed by NIS Elements software in accordance with IS 46-108-01/01 [37]. The software is semi-objective based and uses specific macro for determination of image properties. The user intervention was adopted, and all necessary measurements were obtained stepwise from the fabric real image. The input of the macro is colored image of fabric longitudinal and transverse cross-section, this real image has been processed and transformed to binary system (grey structure), in which we are able to analyze easily where are the fibers in the picture. The test procedure has been showed in Figure 13.

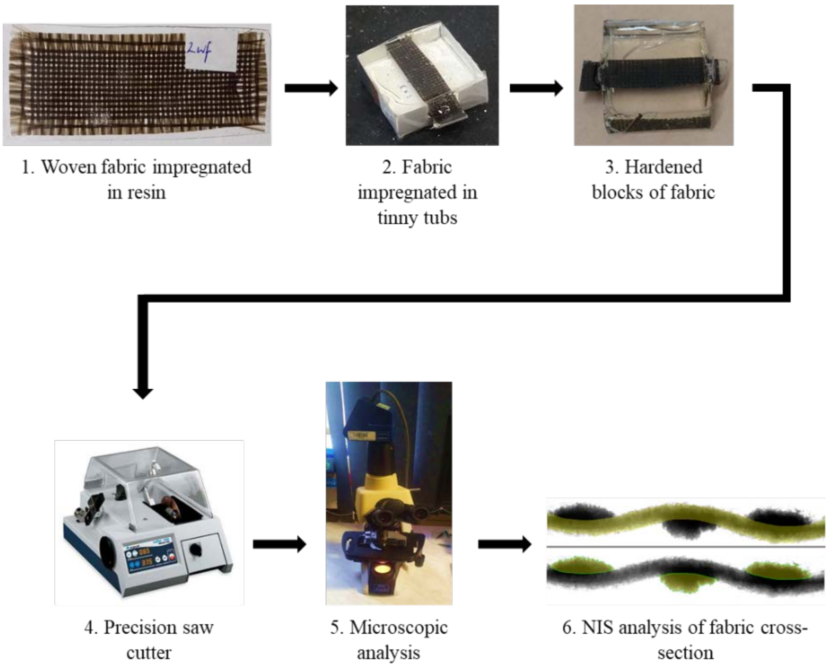


Figure 12. Method for cross-sectional image analysis of woven fabric

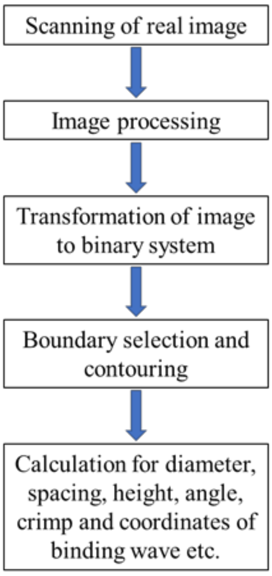


Figure 13. Test procedure for image processing by NIS element software

5.2.3 Shape of yarn in cross-section of woven fabric

Shape (roundness) is a factor which can influence the appearance of the end product of the yarn [38]. It is the ratio of short to long axis of ellipse (circular = 1). Initially the yarn cross-section is assumed to be circular with diameter *‘d’* and incompressible, which becomes a flattened shape after being woven into fabric, due to different stresses on it as shown in Figure 14. After deformation it has the shape having the yarn width (major diameter, in a plane parallel to the fabric surface) *‘a’* and yarn height (minor diameter, in a plane perpendicular to fabric surface) *‘b’*, and usually $a > d$ and $b < d$. It is supposed that the yarn axis is in the middle of *‘a’* and *‘b’*. To calculate the shape factor of the yarn cross-section, the major diameter (a) of yarn in the plane approximately parallel to the fabric surface and minor diameter (b) of yarn in the plane approximately perpendicular to the fabric surface of the elliptical yarn, were measured in each image as shown in Figure 15 [39].

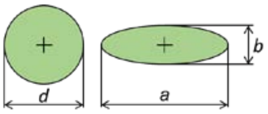


Figure 14. Geometry of yarn cross-section

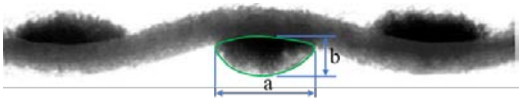


Figure 15. Measuring method of yarn cross-section

6 Summary of the results achieved

6.1 Cross-sectional image analysis of woven fabrics

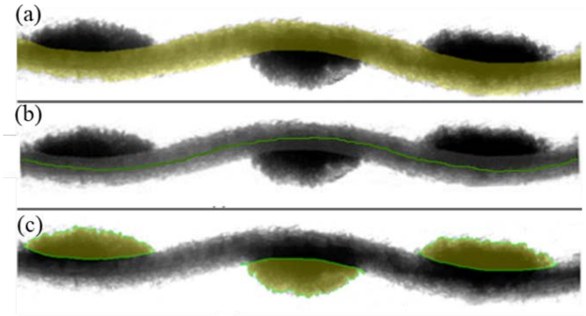


Figure 16. Cross-sectional image of a segmented woven fabric (B1) in NIS software – Overlay image of (a) binding wave, (b) coordinates of center line of binding wave and (c) cross-sections.

The segmented cross-sectional image of a woven fabric (B1) can be seen in Figure 16. It is possible to measure the geometry of the individual fabric cross-section; the diameter of yarns, their deformation, yarn spacing, height of binding wave, the angle of the yarn axis (interlacing angle), the length of the yarn axis in the cross-section of the fabric, the crimp of yarns in the fabric, the real shape of the binding wave through the wave coordinates, and the fabric thickness. The binding wave data of ten samples for each fabric type were obtained and the central line of the average binding wave was calculated. The average binding wave can be seen in Figure 17 for fabric sample (B1).

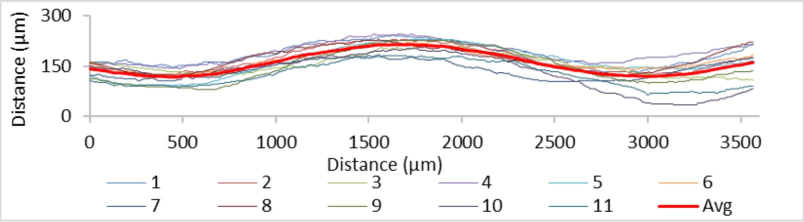


Figure 17. Average binding wave of a single layer fabric (B1) in transverse cross-section

6.2 Evaluation of deformation of thread in cross-section of woven fabric

It is possible to substitute the shape of yarn in the cross-section of woven fabric according to the models of yarn deformation which has been described earlier. The elliptical substitution of yarn (based on Pierce elliptical model) in the cross-sectional image of woven fabric can be seen in Figure 18 and it has been used to calculate the effective values of yarn diameter, which should be used later in Fourier analysis. It is not necessary to use Fourier series analysis for yarn cross-sections but for the binding wave in woven fabric. To analyze the yarn cross-section, it is necessary to use some other theories of prediction. The Fourier series is for periodic function which holds good for the binding wave analysis, while the yarn cross-section is the shape which is given by Pierce's, Kemp's, and Hearl's models.

To study the effect of change in pick density on shape of binding wave and yarn cross-section, it can be observed in Figure 19 that major diameter *(a)* of weft yarn is greater than that of adjacent warp yarns, which means more flatness in weft yarns. The major diameter *(a)* is decreasing from (B1) to (B3) and minor diameter *(b)* is increasing for warp yarn, while it is opposite in case of weft yarns. As the pick density increases, the yarn height *(b)* for warp yarn increases which can result in higher crimp and waviness for the binding wave of weft yarn [40].

Figure 18. Elliptical substitution of the yarn cross-sectional shape in the cross-section of woven fabric

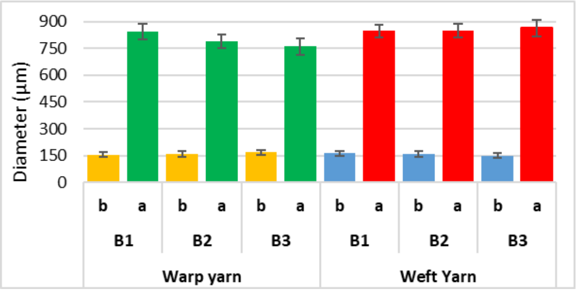


Figure 19. Effect of pick density on major and minor diameter of single layer woven fabrics

6.3 Approximation of binding wave of single layer basalt woven fabrics in cross-section using Fourier series

For the approximation of binding wave in cross-section in single layer of plain woven fabrics by a partial sum of FS with a linear description of the central line of the binding wave, the parameters required are given in Table 2. The linear descriptions of the binding wave in longitudinal and transverse cross-sections for fabric sample (B1) are shown in Figure 20.

Table 2. Input parameters for the mathematical modeling (sample B1)

Yarn count (Tex)	T	132	Height of warp from center (µm)	h_1	109
Warp yarn diameter (µm)	d_1	168	Height of weft from center (µm)	h_2	59
Weft yarn diameter (µm)	d_2	168	Density of warp yarns (1/cm)	D_1	8.15
Mean yarn diameter (µm)	d_s	168	Density of weft yarns (1/cm)	D_2	6.5
Warp yarn spacing (µm)	A	1220	Weft yarn spacing (µm)	B	1539

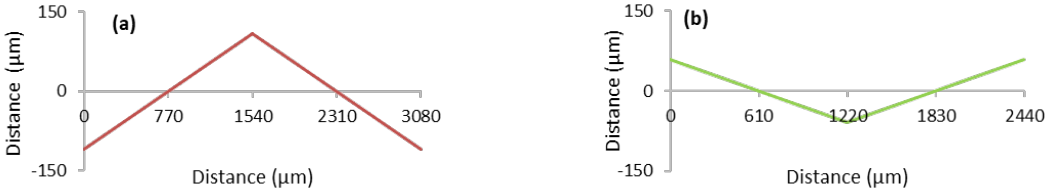


Figure 20. Graphical illustration of linear description of central line of thread in cross-section for sample (B1) in (a) longitudinal, and (b) transverse cross-section of woven fabric

The real cross-section for the sample (B1), its experimental binding waves and FS approximation using equation (29) along longitudinal cross-section can be observed in Figure 21. It can be observed that the approximation done by Fourier series fits well to the experimental binding wave. Each of the binding waves obtained by Fourier series (where $\alpha = 1,2,3,..$) has its spectral characteristic which evaluates the course of the binding wave in terms of geometry, eventual deformation and random changes resulted from the stress of individual threads. The spectral characteristics of the single layer fabric sample (B1) in longitudinal cross-section has been calculated using equation (27) and are also shown in Figure 21. First and second binding waves are identical in the longitudinal and transverse cross-sections. Similarly, the third and fourth binding waves are also identical and so on. The first harmonic component *(A1)* represents the amplitude of the first binding wave, while second harmonic component *(A2)* is the difference between first and second binding wave and as these are identical so the difference between them is zero. In the similar way, the difference between the other binding waves has been calculated which is continuously decreasing. The interlacing in the plain weave is the interlacing with the smallest binding repeat, with only two different interlacing threads in both cross-sections. The height of the first harmonic component *(A1)* also tells us about the deformation of binding wave in comparison with other fabrics. While the third component *(A3)* tells us about the rigidity of woven fabric, when the difference between second and third binding wave is high then this value is more. The higher value of the amplitude of *(A3)* means the fabric *(B1)* is more rigid in longitudinal cross-section. The FS approximation of average binding wave has been performed theoretically using equation (14) and experimentally using equation (29), while their spectral characteristics has been obtained using equation (27) and compared with each other given in Figure 21 as well. It can be observed that our predicted (theoretical) values obtained by Fourier model are close to experimental spectral characteristics values. The difference is in the even number of harmonic components, which is not equal to zero in experimental values. Moreover, it has been observed that just by adding few number of sines and cosines series we can get a better approximation of binding wave. As the Fourier series is an expansion of a periodic function $F(x)$ in terms of an infinite sum of sines and cosines. In the figures $F_1(x)$ (orange line) is the sum of one term of Fourier series, while in $F_3(x)$ (green line) it is the sum of three terms of Fourier series to get better approximation.

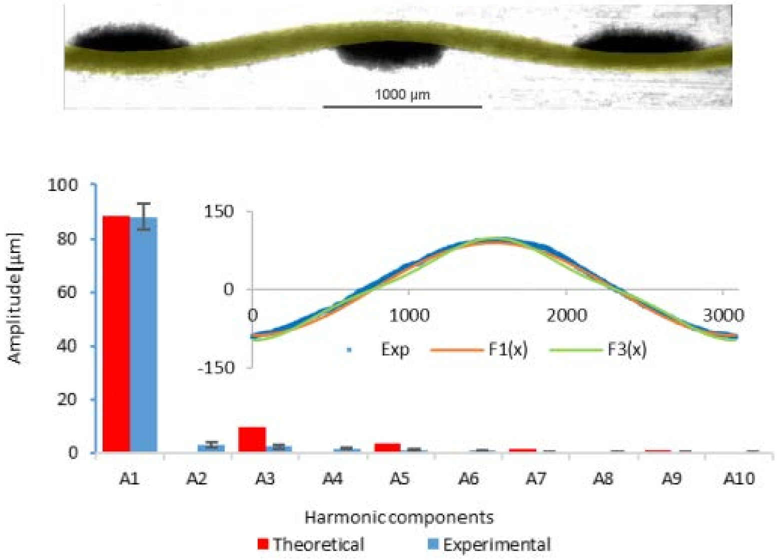


Figure 21. Cross-section of real binding wave of woven fabric, Fourier approximation and spectral characteristics of binding wave in longitudinal cross-section for fabric sample (B1)

Similarly, in Figure 22 and Figure 23 the real cross-sections, Fourier approximation and spectral characteristics of binding wave in longitudinal cross-section for fabric sample (B2) and (B3) can be observed as well. It can also be observed from the Figure 21 to Figure 23 that with the increase in pick density, the deformation in bending wave of warp yarn in longitudinal cross-section is consecutively increasing from (B1) to (B3), while the height of their crimp wave is continuously decreasing, which can also be observed with amplitude of the first harmonic component *(A1)*. The reason for this is that at high pick setting, the weft yarn gets less space to be flat and warp yarn more space, in fabric plane and hence, the binding wave of warp yarn attains more deformation.

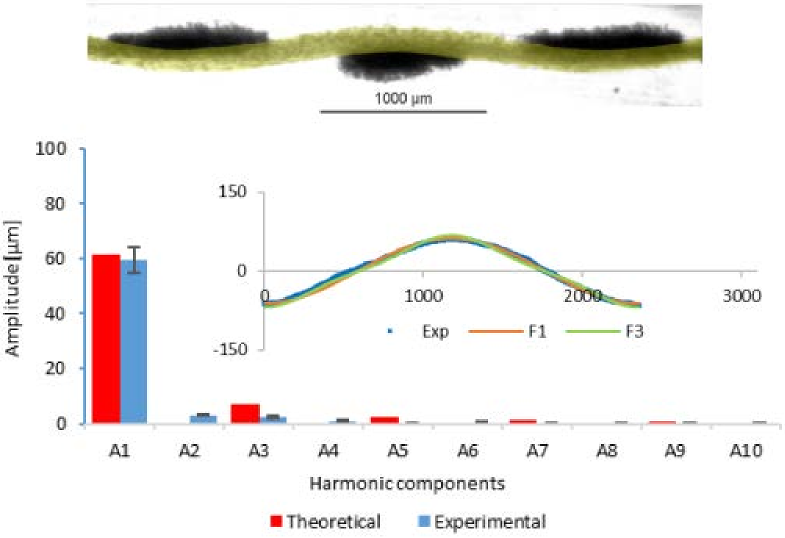


Figure 22. Cross-section of real binding wave of woven fabric, Fourier approximation and spectral characteristics of binding wave in longitudinal cross-section for fabric sample (B2)

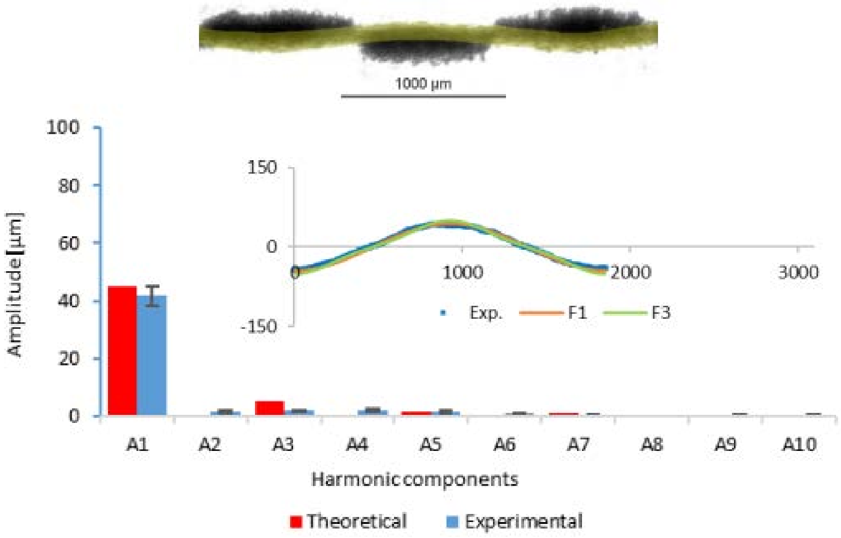


Figure 23. Cross-section of real binding wave of woven fabric, Fourier approximation and spectral characteristics of binding wave in longitudinal cross-section for fabric sample (B3)

The real cross-section for the sample (B1), its experimental binding waves and FS approximation using equation (29) along transverse cross-section can be observed in Figure 24. It can be observed that the approximation done by Fourier series fits well to the experimental binding wave. The difference in amplitude can be analyzed as well, the deformation in transverse cross-section (binding wave of weft) is more as compared to deformation in longitudinal cross-section (binding wave of warp). The spectral characteristics of fabric sample (B1) in transverse cross-section has been calculated using equation (27) and are shown in Figure 24 as well.

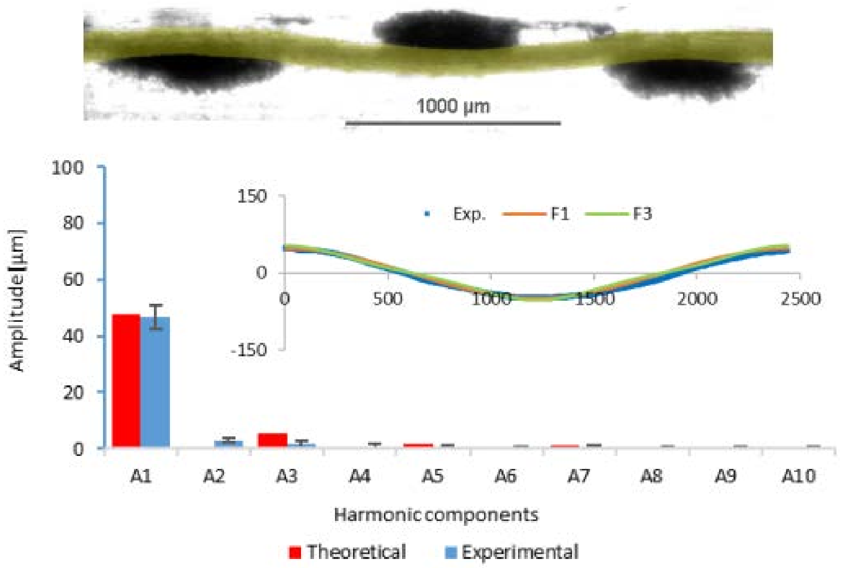


Figure 24. Cross-section of real binding wave of woven fabric, Fourier approximation and spectral characteristics of binding wave in transverse cross-section for fabric sample (B1)

Similarly, in Figure 25 and Figure 26 the real cross-sections, Fourier approximation and spectral characteristics of binding wave in transverse cross-section for fabric sample (B2) and (B3) can be observed as well.

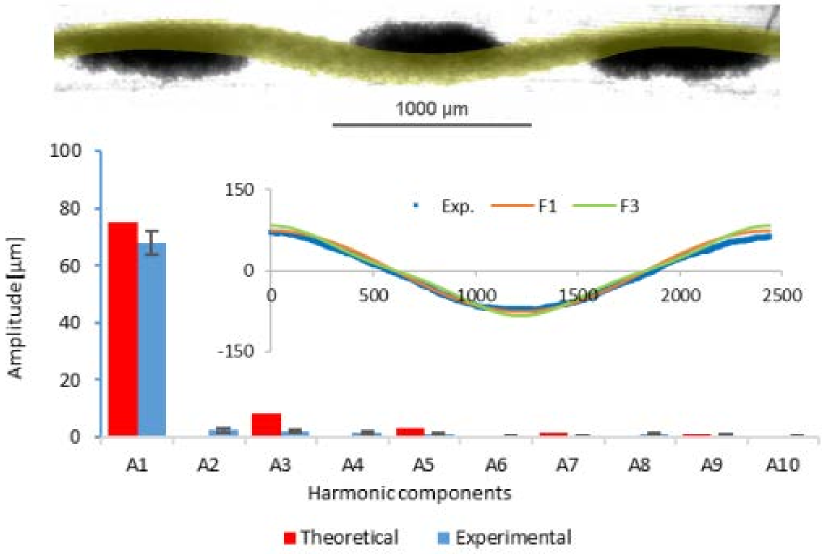


Figure 25. Cross-section of real binding wave of woven fabric, Fourier approximation and spectral characteristics of binding wave in transverse cross-section for fabric sample (B2)

It can also be observed from the Figure 24 to Figure 26 that with the increase in pick density, the deformation in bending wave of weft yarn in transverse cross-section is consecutively decreasing from (B1) to (B3), while the height of their crimp wave is continuously increasing, which can also be observed with amplitude of the first harmonic component *(A1)*.

This is because, at low pick setting, the weft yarn gets more space to be flat in fabric plane and hence, its binding wave attains more deformation.

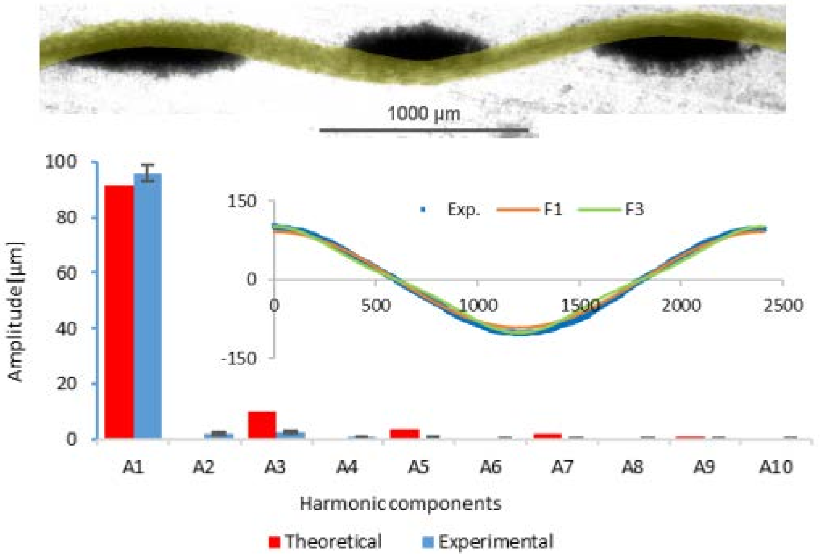


Figure 26. Cross-section of real binding wave of woven fabric, Fourier approximation and spectral characteristics of binding wave in transverse cross-section for fabric sample (B3)

In Figure 27 and Figure 28, the experimental values of binding waves and their spectrum can be observed in longitudinal and transverse cross-section, to analyze their deformation in comparison with each other. It can be analyzed in Figure 27, as the weft density is continuously increasing from (B1) to (B3), it is affecting the width (period) and deformation of the binding wave of warp yarn. It can also be explained by the effect of density in Figure 28 as the interval or density in transverse cross-section is fixed so there is no change in the period of the binding waves of weft yarn, but in the heights (amplitudes). It can be observed, when the density is low (sample B1) the deformation of binding wave of weft yarn is high because of the availability of more yarn spacing, which lets the yarn to deform easily even on small tensions. While at higher densities (sample B3) the spaces between weft yarns are so less that they do not let the weft yarns to get higher deformations. In this case the stresses increase on binding wave of warp yarn and eventually it deforms more. It is the balance of force from the law of action and reaction, the longitudinal and transverse cross-sections complement each other.

The difference in amplitude can be analyzed by the harmonic analyses as well, the deformation in longitudinal cross-section (binding wave of warp) is less for sample (B1) as compared to deformation in transverse cross-section (binding wave of weft). When one yarn gets more deformation then the other yarn connected to it, deforms less as in case of the binding wave of warp yarn. It is called the balance of crimp between warp and weft [41]. Similarly, it can be observed how the amplitude of first harmonic component is decreasing in longitudinal cross-section from (B1) to (B3), while it is increasing in transverse cross-section of woven fabric.

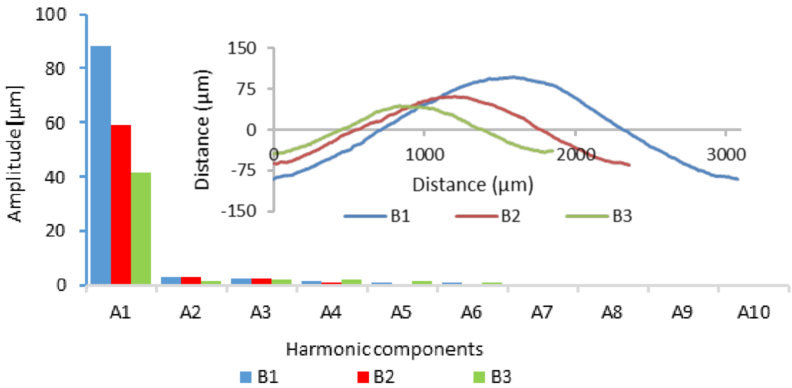


Figure 27. Shape deformation of binding wave in plain woven fabrics (B1-B3) in longitudinal cross-section

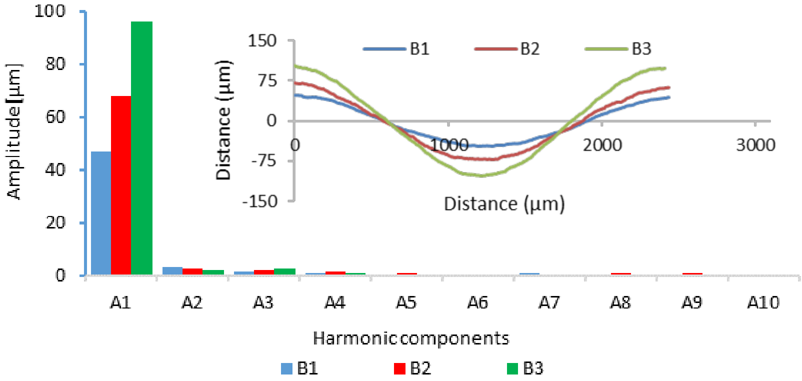


Figure 28. Shape deformation of binding wave in plain woven fabrics (B1-B3) in transverse cross-section

The FS approximation of single layer plain Glass woven fabrics by a partial sum of FS with a linear description of the central line of the binding wave have been performed as well [42]. All the parameters were kept same as of Basalt woven fabric and results are shown in Appendix B. As we have the same Glass and Basalt material and their linear density is also almost same, so we have obtained the similar results of approximation and harmonic analysis. Some small changes in deformation are given by the irregularity and non-uniformity of the structures.

6.4 Approximation of binding wave of two layer stitched basalt woven fabrics in cross-section using Fourier series

The fabric design and their real cross-sectional images of binding wave of two layer woven fabrics (B4-B7) with varying stitching distance are shown in Figure 29. The binding wave in two layer stitched plain woven fabric can be analyzed and approximated in different methods. These methods are described further.

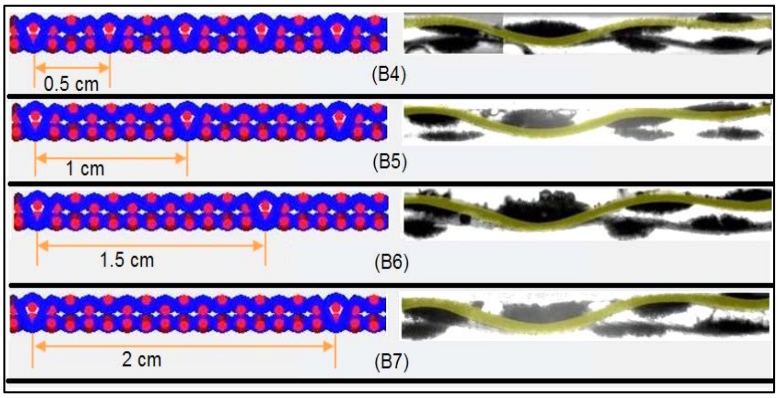


Figure 29. Cross-sectional image of binding wave of two layer woven fabrics (B4-B7) in longitudinal cross-section with varying stitching distance along warp thread (left side: theoretical simulation of cross-section, right side: real cross-section of woven fabric)

6.4.1 FS approximation of stitching section of binding wave of two layer stitched woven fabric in cross-section

The geometry of two layer stitched woven fabrics with plain weave has been divided into stitching and non-stitching section as shown in Figure 8. When the number of stitching points decreasing from (B4) to (B7), it increases the forces inside the thread during weaving. So, we need to analyze that whether the deformation is different at stitching points or not. The linear description of binding wave in two layer stitched fabric can be observed in Figure 8 as well. This description allows us to evaluate the warp and weft threads in the interlacing at stitching sections.

The experimental binding wave of samples (B4-B7) and its approximation using equation (14) in longitudinal cross-section can be observed in Figure 30 to Figure 33 for stitching sections. It also contains the spectral characteristics of binding wave. Each of the binding waves obtained by Fourier series (where $\alpha = 1,2,3,..$) has its own spectral characteristic which evaluates the course of the binding wave in terms of geometry, eventual deformation and random changes, resulted from the stress of individual threads. It can be observed in all figures that the approximation done by Fourier series fits well to the experimental binding wave. The deformation is slightly changing in stitching sections of binding wave in two layer woven samples (B4-B7), which can be analyzed accurately by the harmonic analysis. The shape of the binding wave is not similar in all cases. The spectral characteristics obtained by theoretically and experimentally approximated binding wave has been calculated using equation (27) for stitched sections. The first harmonic component *(A1)* represents the amplitude of the first binding wave, second harmonic component *(A2)* is the difference between first and second binding wave and so on. The difference between the binding waves has been rapidly decreasing, which shows that it is not necessary to use higher number of harmonic components to get the better approximation.

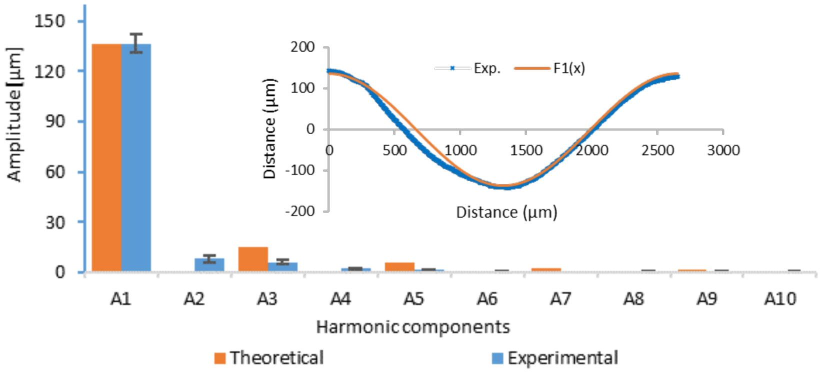


Figure 30. Fourier approximation and spectral characteristics of binding wave in stitched section of binding wave for sample (B4)

The amplitude of the first harmonic component *(A1)* for the first binding wave is increasing in all four cases from (B4) to (B7). All the input parameters are same for these fabric samples except of the stitch distance, which explains the effect of stitch distance in two layer woven structure is significant on the deformation of the binding wave. As the amplitude of first harmonic component *(A1)* is increasing from (B4) to (B7), it means the deformation is decreasing in stitching section. In other words, it can also be said that the woven fabric sample (B4) possess maximum deformation of binding wave, while woven fabric sample (B7) possess less deformation of binding wave as compared to other fabric samples. This change in amplitude or deformation is due to varying number of stitch points in all woven structures. When there are more number of stitch points and stitch distance is short, then more length of stitching yarn will be required by the same input weaver beam. Therefore, the tension on stitching yarn increases, which result in higher deformation of binding yarn in the stitching area. The second harmonic component *(A2)* gives the information about the rigidity of woven fabric, when the difference between first and second binding wave is higher, then this value is high. The amplitude of harmonic components after *(A1)* is not high which shows good approximation and only few terms will be required for better approximation.

It can also be observed in Figure 30 to Figure 33 that our predicted (theoretical) values of harmonic components obtained by Fourier model are in accordance with the experimental spectral characteristics values. First and second binding waves obtained using theoretical model are identical in the longitudinal cross-section. Similarly, the third and fourth binding waves are also identical and so on. As these binding waves are identical, so the difference between them is zero in spectral characteristics. In the similar way, the difference between the other binding waves has been calculated, which is continuously decreasing.

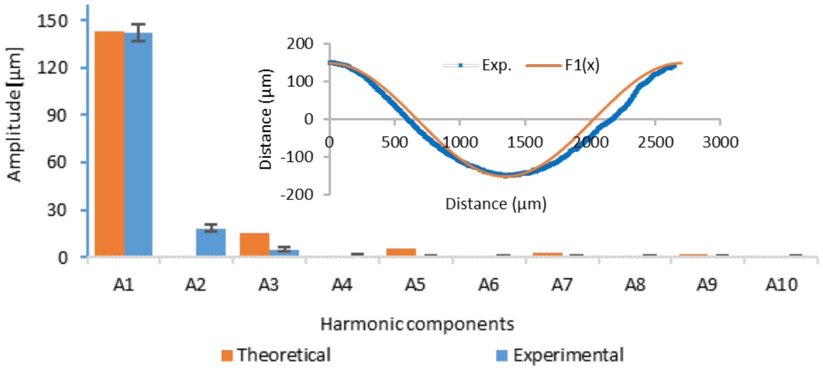


Figure 31. Fourier approximation and spectral characteristics of binding wave in stitched section of binding wave for sample (B5)

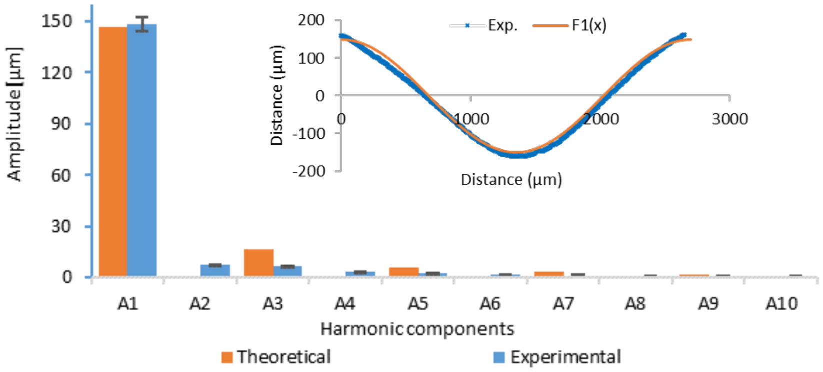


Figure 32. Fourier approximation and spectral characteristics of binding wave in stitched section of binding wave for sample (B6)

In Figure 34, the binding wave for two layer woven fabrics at stitching area obtained by the cross-sectional image analysis of real fabric can be observed. It can be analyzed that there is a slight change in the course of binding wave in all woven fabric samples, which depicts different amplitudes and deformations for different fabric structures at stitching area. This difference has been calculated and explained by the harmonic analysis of experimental binding wave as well and can be observed with a slight increase in amplitude of first harmonic component from (B4) to (B7).

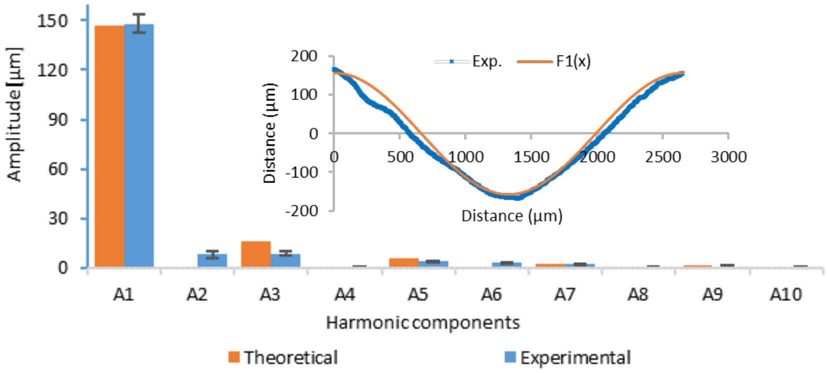


Figure 33. Fourier approximation and spectral characteristics of binding wave in stitched section of binding wave for sample (B7)

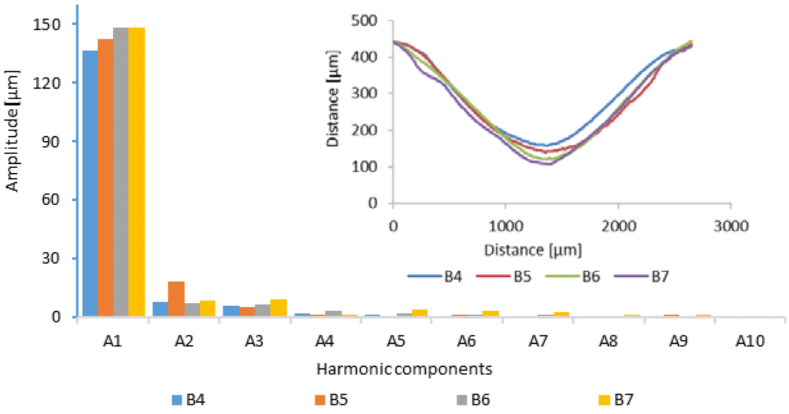


Figure 34. Shape deformation of binding wave and spectral characteristics of binding wave of two layer woven fabrics at stitching area

6.4.2 FS approximation of whole binding wave of two layer stitched woven fabric in cross-section

It is not possible to use the approximation performed on separate parts of binding wave for the evaluation of properties of whole binding wave because we need the information about the whole interlacing, so we are approximating the whole binding wave as well. The linear description by means of straight lines for two layer stitched woven fabric has been shown in Figure 8 in the fabric geometry. The FS approximation of two layer stitched woven fabrics has been performed using equation (19). The linear description of the binding wave in longitudinal cross-sections for fabric sample (B4) is shown in Figure 35.

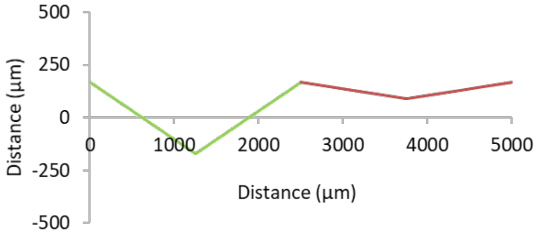


Figure 35. Graphical illustration of linear description of central line of thread in cross-section of two layers stitched woven fabric for sample (B4) in longitudinal cross-section

The real cross-section, experimental binding waves for the sample (B4), its approximation and spectral characteristics can be observed in Figure 36. It can be observed that the approximation done by Fourier series fits well to the experimental binding wave after certain number of components and our theoretical model for two layer stitched woven fabric samples holds good with the experimental binding wave.

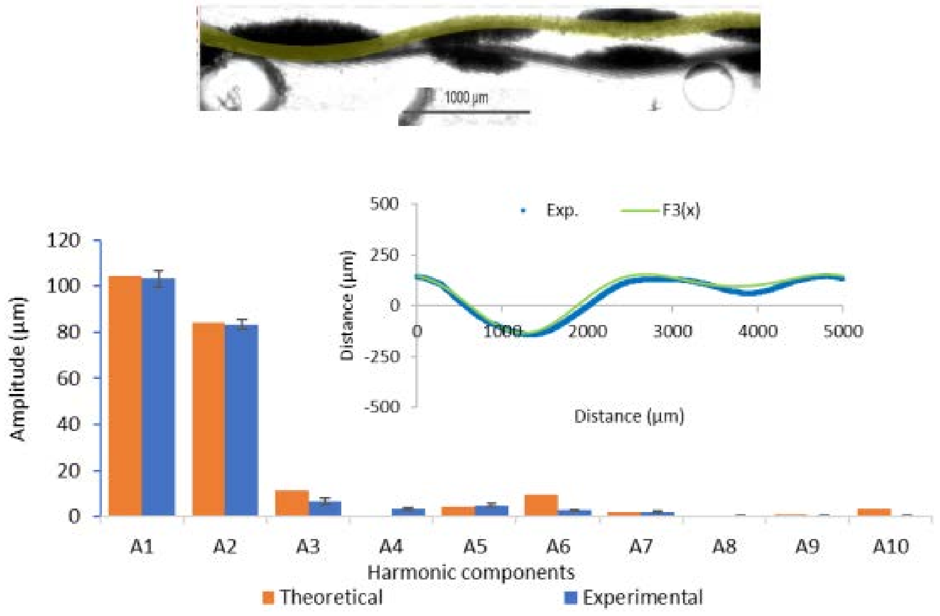


Figure 36. Cross-section of real binging wave of woven fabric, Fourier approximation and spectral characteristics of binding wave (B4) in longitudinal cross-section

The spectral characteristics obtained by theoretically and experimentally approximated binding wave has been calculated using equation (27) for two layer stitched woven fabric (B4). It can be observed in Figure 36 that there is not big difference in the spectral characteristics obtained by theoretical and experimental data. It can also be observed the that the amplitude of first harmonic component *(A1)* is different from the one obtained by FS analysis of stitched portion, as shown in Figure 34, as these values are for both stitched and non-stitched sections. The amplitude of second harmonic component *(A2)* is quite high as the difference between first and second binding wave is high, while the amplitude of remaining components is not so high This explains that a better approximation and fitting by Fourier model can be obtained by using just few number harmonic components and in this case, it is $F_3(x)$.

6.5 Influence of plain weave repeat in non-stitched part of binding wave of two-layer stitched woven fabric in cross-section using Fourier series

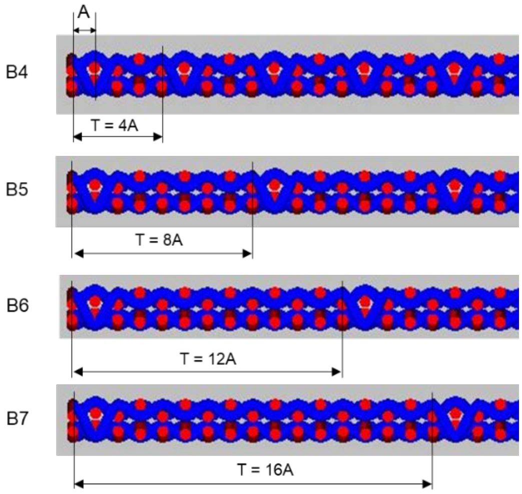


Figure 37. Geometry of two layer woven fabric samples (B4-B7) with varying repeat size

In this chapter we want to present influence of number of repeats of plain weave in binding wave on the spectral characterization of Fourier spectrum. For modelling of geometry of cross-section of woven fabric, the Fourier series is still used with straight lines description of central line of the binding wave. For experimental part of work the fabric design of two layer woven fabrics (B4-B7) with varying stitching distance (period) and repeat size are shown in Figure 37. In the sample (B4) the repeat size is smallest which is one-time, while in (B5) to (B7) it is three, five and seven-times respectively. So, we have a possibility to use this plain weave by ‘n’ times and we can assume that which shape and spectral characteristics we will obtain. The non-stitching section which is just next to the stitching section, can be continuous depending upon the distance between two stitch points or repeat size. There will be higher elongation in fabric when the stitch points per meter are high and this can be used as reinforcement in composites where we need higher deformation instead of rigidity. So, it is necessary to understand the effect of stitch distance or repeat size by FS approximation and spectrum analysis.

The linear description for two layer fabrics has already been describes in Figure 8 and Figure 9 in the fabric geometry. This description allows us to evaluate the warp and weft threads in the interlacing at stitching and non-stitching sections for a complete binding repeat. For the approximation of a two layer woven fabric (B4-B7) by FS, the period of the periodic function has been taken *as* $P = T = 4A, 8A, 12A, 16A$ respectively.

The FS approximation of two layer stitched woven fabrics has been performed using equation (19). The approximation of the complete repeat of all woven fabric samples (B4-B7) and their spectral characteristics can be observed in the Figure 38 to Figure 41. It can be observed that the approximation done by Fourier series is in accordance with the shape of the binding wave in fabric sample after certain number of components and our theoretical model for two layer stitched woven fabric samples holds good for different repeat sizes as well. It is continuous and can be applied to bigger repeat sizes. The woven fabric (B4) has the smallest repeat size as it contains only one-time plain woven fabric in non-stitching section, while the sample (B5) contains three-times, sample (B6) five-times and sample (B7) holds seven-times plain woven fabric in non-stitching section. Fourier series is an expansion of a periodic function $F(x)$ in terms of an infinite sum of sines and cosines. In the Figure 38 $F_3(x)$ is the sum of five terms of Fourier series to get better approximation. It can be observed in Figure 38 to Figure 41 that when the repeat size is increasing from (B4) to (B7), the number of binding waves required to get a better approximation as per woven structure are also increasing, so the repeat size has direct relation with the number of binding or crimp waves.

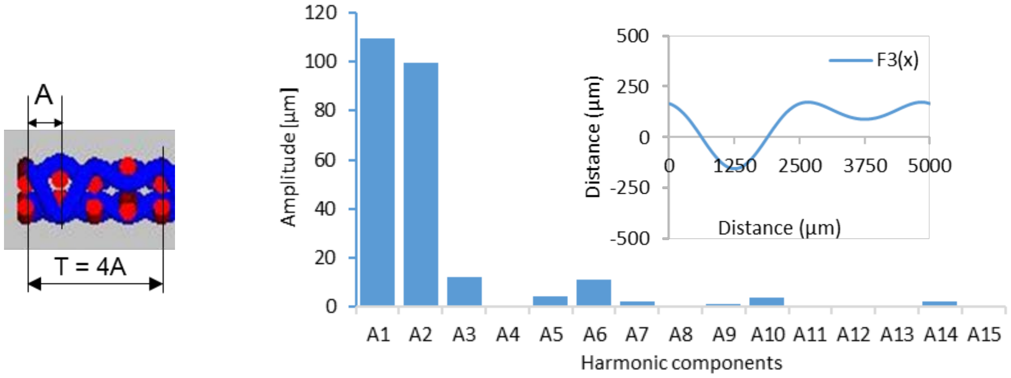


Figure 38. Graphical illustration of geometry of cross-section for one-time repeat of plain woven fabric in non-stitching section, Fourier approximation and spectral characteristics of complete repeat of binding wave (B4) in longitudinal cross-section

Moreover, it can be analyzed accurately by the harmonic analysis. Each of the binding waves obtained by Fourier series (where $\alpha = 1,2,3, ..$) has its own spectral characteristic which evaluates the course of the binding wave. The spectral characteristics of all the two layer fabric sample (B4-B7) in longitudinal cross-section has been calculated using equation (27). The first harmonic component *(A1)* represents the amplitude of the first binding wave, while second harmonic component *(A2)* is the difference between first and second binding wave. In the similar way, the difference between the other binding waves has been calculated and represented in figures for fifteen harmonic components. Moreover, it has also been observed that just by adding few number of sines and cosines series we can get a better approximation of binding wave.

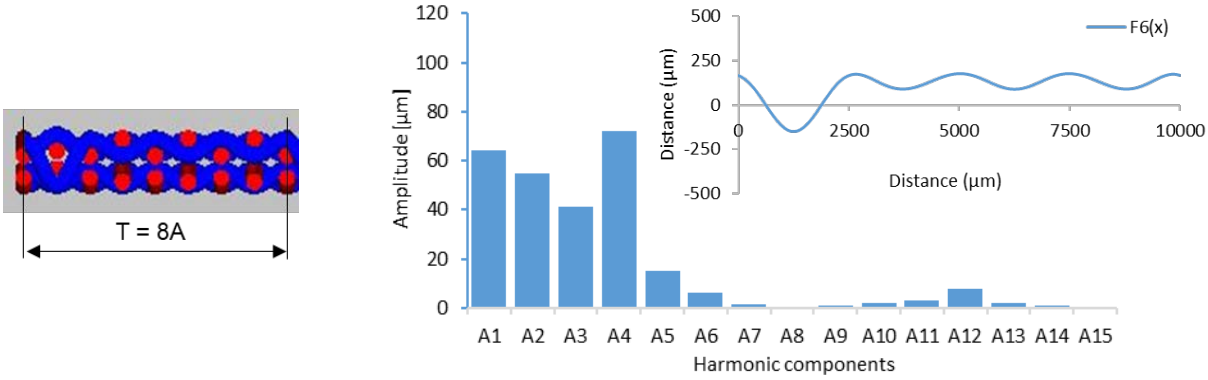


Figure 39. Graphical illustration of geometry of cross-section for three-times repeat of plain woven fabric in non-stitching section, Fourier approximation and spectral characteristics of binding wave (B5) in longitudinal cross-section

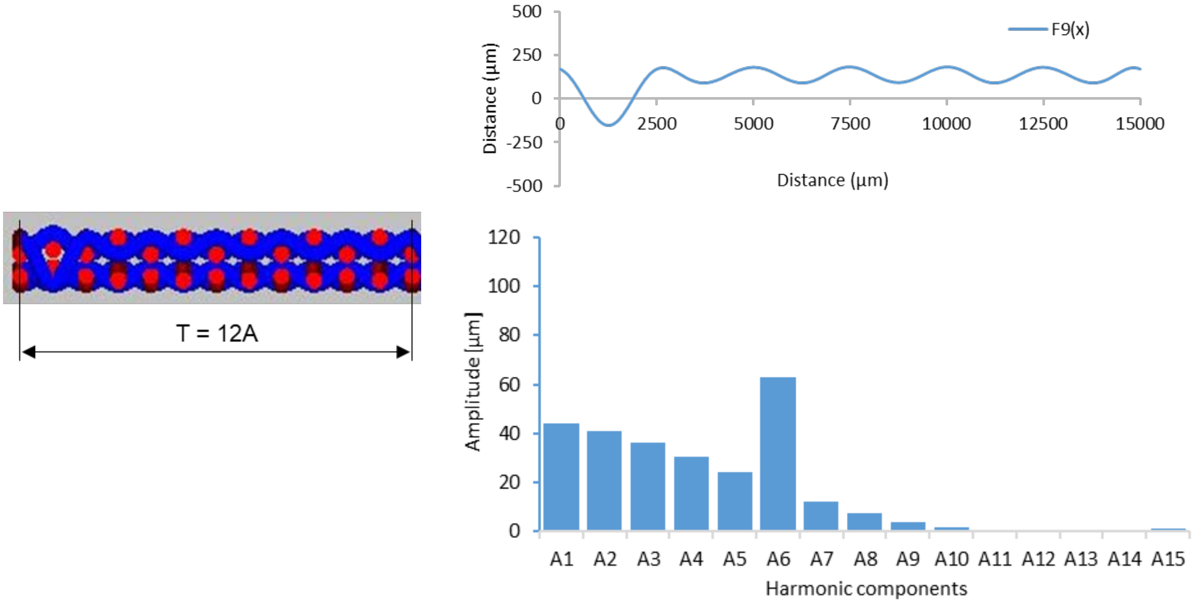


Figure 40. Graphical illustration of geometry of cross-section for five-times repeat of plain woven fabric in non-stitching section, Fourier approximation and spectral characteristics of binding wave (B6) in longitudinal cross-section

It can be observed in Figure 38 to Figure 41 in the harmonic analysis that the amplitude of second component *(A2)* is quite high for woven sample (B4), fourth component *(A4)* for woven sample (B5), sixth component *(A6)* for woven sample (B6) and eighth component *(A7)* for woven sample (B7). After this highest amplitude the approximated crimp wave holds good with the sample crimp wave. We are getting this highest amplitude component after the exact number of plain weave units for each woven fabric sample. So, it can be concluded that when the repeat size is increasing from (B4) to (B7), the number of binding waves required to get a better approximation as per woven structure are also increasing, hence the repeat size is directly proportional to the number of harmonic components.

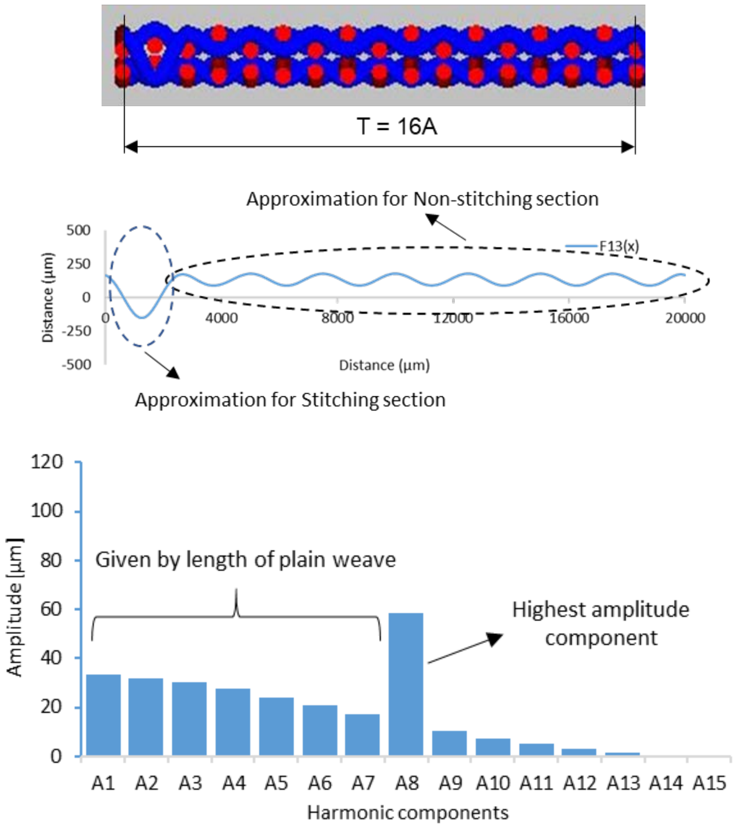


Figure 41. Graphical illustration of geometry of cross-section for seven-times repeat of plain woven fabric in non-stitching section, Fourier approximation and spectral characteristics of binding wave (B7) in longitudinal cross-section

7 Evaluation of results and new finding

The following conclusions can be drawn from this study.

- The work is focused on mathematical expression and description of geometry of binding wave of single as well as two-layer stitched woven fabric using Fourier series. For mathematical definition of binding wave as an input function f(x) in Fourier series, the linear description of the central line of the binding wave was used. Based on verification of model with real binding wave, the validity of the model is confirmed and verified.

- Fourier series analysis describe the whole binding wave repeat contrary to present models which describes only one interlacing point. Using Fourier series modelling, we can obtain not only the description of geometry of binding wave (length of binding wave or crimp of threads) but also spectral characterization for analyzing the individual components, which can react on deformation of the shape of binding wave.

- The final shape of binding wave of two layer stitched woven fabric is given by the definition of number of repeats of plain weave in interlacing in cross-section. The influence of repeats of plain weave in binding wave on the spectral characterization of Fourier spectrum was evaluated.
- FS approximation can be used for analysis of cross-section as well as the deformation of shape of binding wave in the single layer plain woven fabric (which is possible to use as reinforcement structure) and also for two layer plain woven structures where we have connections of the individual layers by the varying stitching points. The model has been validated with the experimental analyses of binding wave in cross-section of single layer and two layer stitched woven fabric.
- The effect of stitch distance (repeat size) in two layer woven structure has been analyzed in stitching section. The amplitude of harmonic component is increasing from (B4) to (B7). The fabric with more number of stitching points (B4) retains more deformation.
- Experimental analysis shows that the approximation done by the proposed theoretical model (idea) for description of geometry of cross-section of woven multifilament fabric structures (single and two layer stitched woven fabrics with plain weave) is in accordance with experimental spectrum. Theoretical model for description of mutual interlacing of threads in two layer stitched woven structure, it is possible to use it as analytical prediction of geometry of woven structure, and shape of the binding wave.
- By predicting the geometry of the binding wave in two layers stitched woven fabric, it is possible to use it for the calculation of basic properties of woven fabric (length of threads in binding wave, crimp of threads, thickness of woven fabric, etc.).
- The model for description of geometry of binding wave in cross-section can be extended for the other weaves such as twill or satin weaves with longer floats, bigger repeat size and more number of layers. The spectrum will be different in that case.
- The deformation of the shapes of binding waves of the individual layers of the multilayer woven fabrics can be compared with the deformation of the binding waves in the individual layers in composites.

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9 List of papers published by the author

Publications in journals

1. **Ahmad Z**., Sirková B., Ahmad S., Naeem M.S., Hassan S.Z., “Effect of material and stitching on tensile properties of woven fabrics”, IOP Conference Series: Materials Science and Engineering, 2018, 414(1): 012049, ISSN 1757-8981.
2. **Ahmad Z.**, Eldeeb M., Iqbal S., and Mazari, A., “Effect of yarn structure on cover factor in woven fabrics”, Industria Textila, 2018, 69(3), p. 197-201, ISSN:1222-5347 [Thompson ISI/Scopus].
3. **Ahmad Z.**, and Sirková B., “Tensile behavior of Basalt and Glass single layer and multilayer woven fabrics”, Journal of Textile Institute, 2017, 109(5), p. 686-694, ISSN: 0040-5000 [Scopus Database].
4. **Ahmad Z.**, Sirková B., and Eldeeb M., “Yarn cross-sectional deformation in woven fabric”, Vlakna a Textil, 2016, 23(4), p. 36-41, ISSN: 1335-0617 [Scopus Database].

Articles under review

1. **Ahmad Z.**, and Sirková B., “Modelling of multifilament woven fabric structure using Fourier series”, Journal of Textile Institute, ISSN: 0040-5000 (under review).
2. **Ahmad Z.**, and Sirková B., “Analysis of mutual interlacing of threads in multifilament single layer and two layer woven fabric structure using Fourier series”, Journal of Textile Institute, ISSN: 0040-5000 (under review).
3. **Ahmad Z.**, and Sirková B., “Modelling of reinforcement two layer stitched woven fabric structure intended for composites”, Composites Part B: Engineering, ISSN: 1359-8368 (under review).

Publications in International Conferences

1. **Ahmad Z**., Sirková B., and Aboalasaad A.R.R., “Yarn deformation in multifilament single layer woven structures using Fourier series”, AUTEX, Istanbul, Turkey 2018, ISBN 978-961-6900-17-1.
2. Zubair M., **Ahmad Z**., Javaid M.U., Drean J.Y., and Mathieu D., “Influence of fabric architecture and material on physical properties of 3D multilayer woven fabrics”, In: 9th Central European Conference (proceedings). Liberec: Technical University in Liberec, 2017. ISBN 978-80-7494-355-3.

3. **Ahmad Z.**, Sirková B. and Eldeeb M., “Influence of weft setting on shape of yarn cross section in woven.” In: Světlanka Workshop (Proceedings), Rokytnice nad Jizerou: Světlanka, 2015-08-14. ISBN 978-80-7494-229-7.
4. **Ahmad Z.**, Nazir I., Ibrahim S., Iqbal S., and Naeem M.S., “Effect of weave design on comfort properties of fabric made of Bamboo yarn” In: 20th conference STRUTEX (proceedings). Liberec: Technical University in Liberec, 2014. ISBN 978-80-7494-139-9.

Research Project

1. Leader of the student grant completion (SGS) project 2016 (project no.21153), titled, “3D construction and structure of woven fabric”, Faculty of Textile Engineering, Technical University of Liberec, Czech Republic.

Curriculum Vitae

Personal information

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E-mail	zuhaib_240@yahoo.com
Nationality	Pakistan
Date of Birth and place	02 May 1985, Rawalpindi, Pakistan
Gender	Male

Research/work experience

• Textile Technologies and Structures, Faculty of Textile Engineering, Technical University of Liberec, Liberec, Czech Republic.	PhD Scholar	2013 – present
• Fabric Manufacturing Department, National Textile University, Faisalabad, Pakistan	Lecturer	2011 – present (on study leave)
• Weaving L10, Crescent Textile Mills, Ltd. Faisalabad, Pakistan	Senior Assistant Manager	2008 –2010

Academic profile

• University of Boras, Boras, Sweden.	The Swedish School of Textiles	M.Sc.	2011
• National Textile University, Faisalabad, Pakistan	Fabric Manufacturing Department	B.Sc. Textile Engineering	2008

Impact factor journal publications

1. **Ahmad Z**., Sirková B., Ahmad S., Naeem M.S., Hassan S.Z., “Effect of material and stitching on tensile properties of woven fabrics”, IOP Conference Series: Materials Science and Engineering, 2018, 414(1): 012049, ISSN 1757-8981.

2. Naeem M.S., Hassan S.Z, Javed Z., Ramzan B., Rasheed A., **Ahmad Z.**, Javed S., “Electromagnetic shielding effectiveness of high loft activated carbon web prepared by using acrylic waste”, IOP Conference Series: Materials Science and Engineering, 2018, 414(1): 012049, ISSN 1757-8981.
3. **Ahmad Z.**, Eldeeb M., Iqbal S., and Mazari, A., “Effect of yarn structure on cover factor in woven fabrics”, Industria Textila, 2018, 69(3), p. 197-201, ISSN:1222-5347 [Thompson ISI/Scopus].
4. Azeem M., **Ahmad Z.**, Wiener J., Fraz A., Siddique H.F., and Havalka A., “Influence of weave design and yarn types on mechanical and surface properties of woven fabric”, Fibres & Textiles in Eastern Europe, 2018, 26, 1(127), p. 42-45, ISSN:1230-3666, [Thompson ISI/Scopus].
5. **Ahmad Z.**, and Sirková B., “Tensile behavior of Basalt and Glass single layer and multilayer woven fabrics”, Journal of Textile Institute, 2017, 109(5), p. 686-694, ISSN: 0040-5000 [Scopus Database].
6. Iqbal S., Eldeeb M., **Ahmad Z.**, and Mazari A., “Comparative study on yarn and knitted fabric made from Open end and Rieter airjet spun system”, Journal of Textile & Apparel/Tekstil ve Konfeksiyon, 2017, 27(3), p. 234-240, ISSN: 1300-3356 [Thompson ISI/Scopus].
7. Mangat, A.E., Hes, L. Bajzik, V., and **Ahmad, Z.**, “Influence of air flow direction on thermal resistance and water vapor permeability of Rib knit fabrics”, Journal of Textile & Apparel/Tekstil ve Konfeksiyon, 2017, 27(1), p. 32-37, ISSN: 1300-3356 [Thompson ISI/Scopus].
8. **Ahmad Z.**, Sirková B., and Eldeeb M., “Yarn cross-sectional deformation in woven fabric”, Vlakna a Textil, 2016, 23(4), p. 36-41, ISSN: 1335-0617 [Scopus Database].

Conference publications

1. **Ahmad Z**., Sirková B., and Aboalasaad A.R.R., “Yarn deformation in multifilament single layer woven structures using Fourier series”, AUTEX, Istanbul, Turkey 2018, ISBN 978-961-6900-17-1.
2. Naeem M.S., **Ahmad Z.**, Javed Z., Rasheed A., Ramzan B. and Abid H.A., "Removal of Acid Red from Aquous Media using Activated Carbon from Acrylic waste", Emerging Trends in Knitting, Venue: National Textile University, Faisalabad, Pakistan, 2018, ISBN 978-969-7549-03-0.
3. Zubair M., **Ahmad Z**., Javaid M.U., Drean J.Y., and Mathieu D., “Influence of fabric architecture and material on physical properties of 3D multilayer woven fabrics”, In: 9th Central European Conference (proceedings). Liberec: Technical University in Liberec, 2017. ISBN 978-80-7494-355-3.
4. Naeem S., Javed S., Baheti V., Militky J., **Ahmad Z.**, and Behera P., “Effect of temperature, heating rate and holding time on the properties of Carbon web made from acrylic waste” In: 21st conference STRUTEX (proceedings). Liberec: Technical University in Liberec, 2016. ISBN 978-80-7494-269-3.
5. **Ahmad Z.**, Sirková B. and Eldeeb M., “Influence of weft setting on shape of yarn cross section in woven.” In: Světlanka Workshop (Proceedings), Rokytnice nad Jizerou: Světlanka, 2015-08-14. ISBN 978-80-7494-229-7.
6. **Ahmad Z.**, Nazir I., Ibrahim S., Iqbal S., and Naeem M.S., “Effect of weave design on comfort properties of fabric made of Bamboo yarn” In: 20th conference STRUTEX (proceedings). Liberec: Technical University in Liberec, 2014. ISBN 978-80-7494-139-9.
7. Javaid M.U., **Ahmad Z.**, Iqbal S., Naeem M.S., “Viscose Fiber Strength and Degree of Polymerization”, In: First International Young Engineers Confrence, At University of

Engineering and Technology, Lahore, April 2014

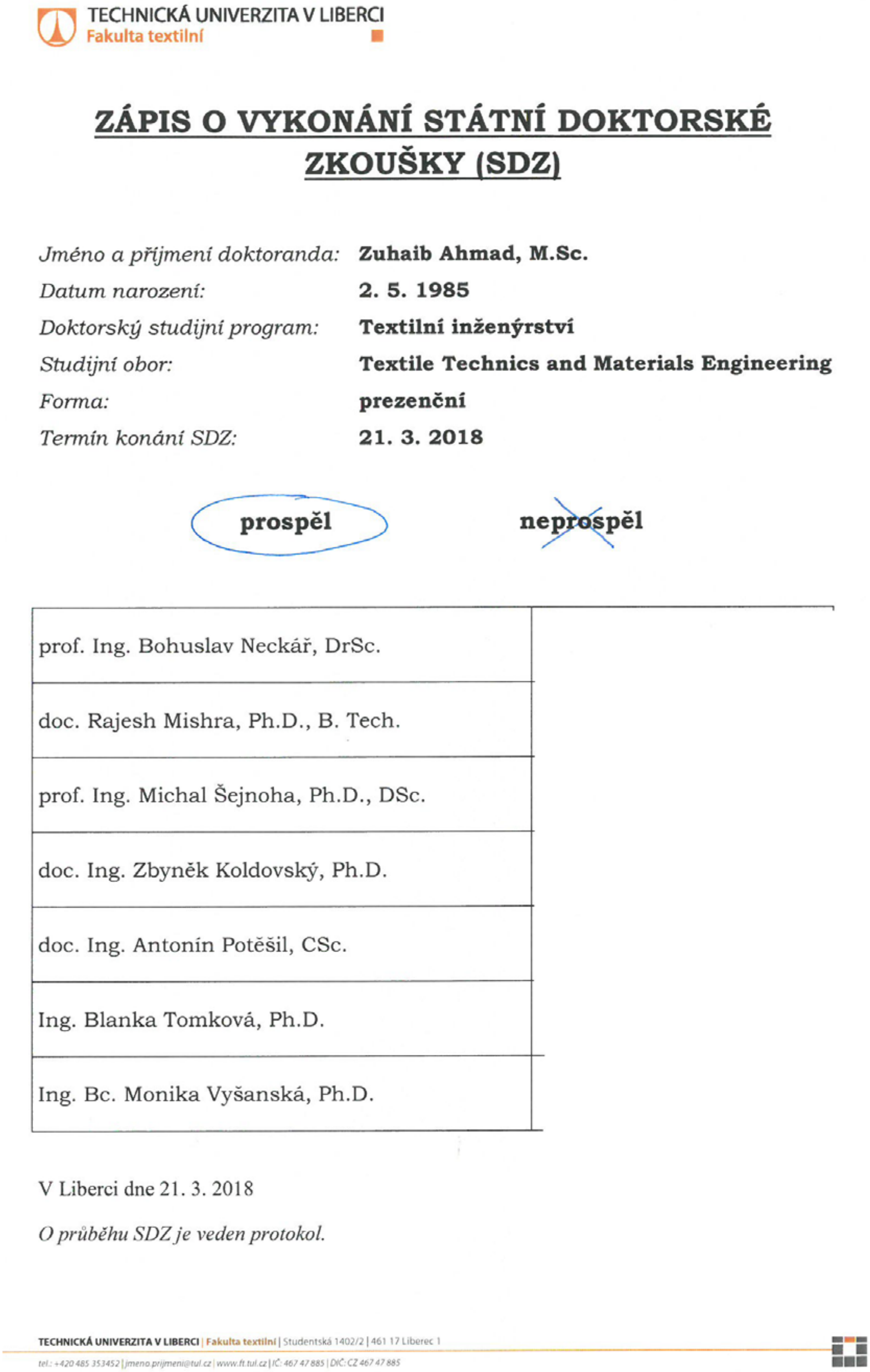
Research projects

1. Member of the student grant completion (SGS) project 2017 (project no.21198), titled, “Preparation of modified carbon sorbents”, Faculty of Textile Engineering, Technical University of Liberec, Czech Republic.
2. Leader of the student grant completion (SGS) project 2016 (project no.21153), titled, “3D construction and structure of woven fabric”, Faculty of Textile Engineering, Technical University of Liberec, Czech Republic.
3. Member of the student grant completion (SGS) project 2015 (project no.21086), titled, “Effect of plying process on air jet yarn properties”, Faculty of Textile Engineering, Technical University of Liberec, Czech Republic.

Other professional activities

1. Two-month Erasmus+ traineeship from February 2017 to April 2017 at University of Huate Alsace, Mulhouse, France.
2. Member of Pakistan Engineering Council. Pakistan.
3. Member of National Textile University Alumni Association. Pakistan.

Record of the state doctoral exam



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Doktorský studijní program: **Textilní inženýrství**
Studijní obor: **Textile Technics and Materials Engineering**
Forma: **prezenční**
Termín konání SDZ: **21. 3. 2018**

[✓ **prospěl**] ~~**neprospěl**~~

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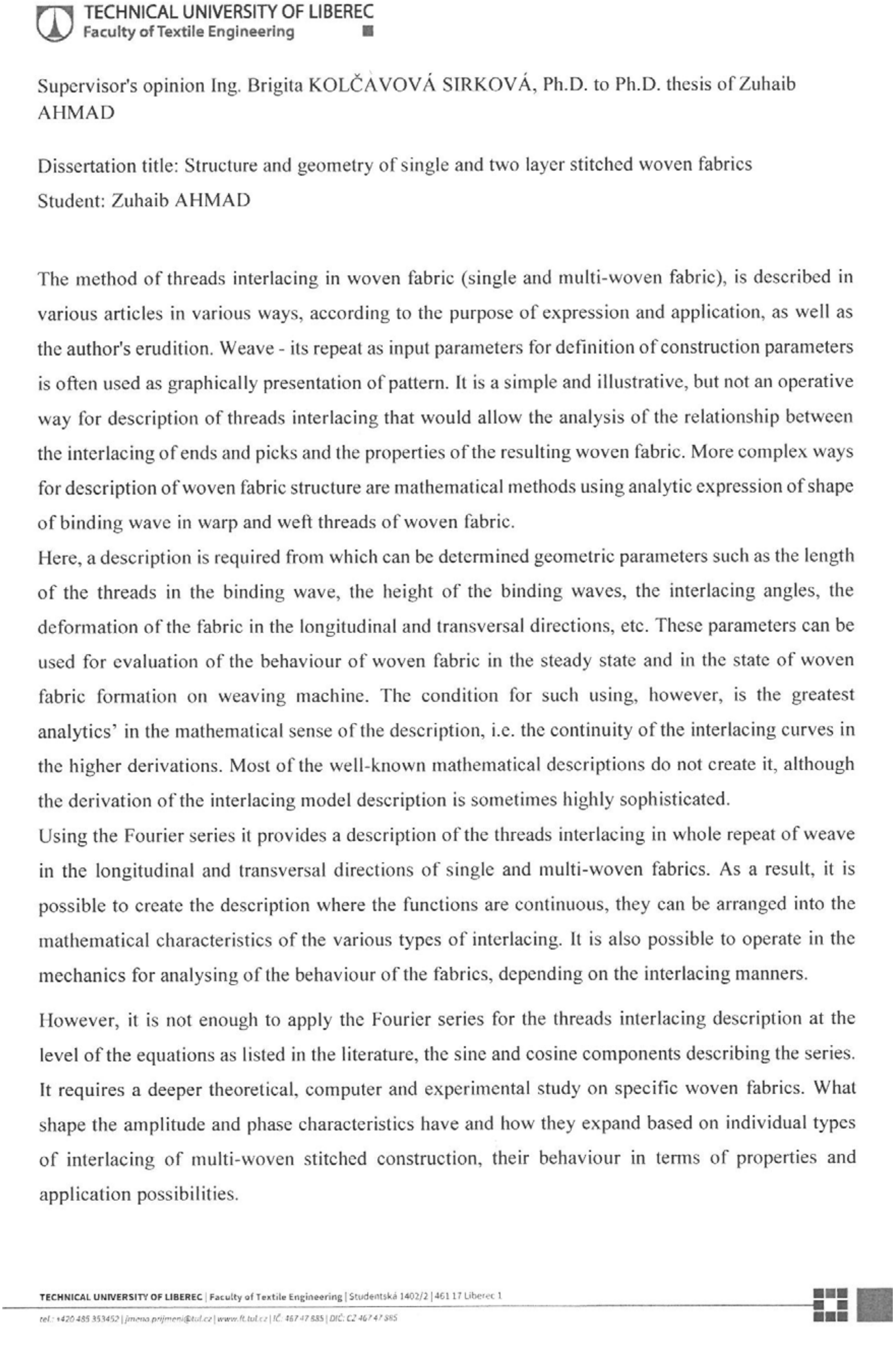
Recommendation of the supervisor



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Supervisor's opinion Ing. Brigita KOLČAVOVÁ SIRKOVÁ, Ph.D. to Ph.D. thesis of Zuhaib AHMAD

Dissertation title: Structure and geometry of single and two layer stitched woven fabrics
Student: Zuhaib AHMAD

The method of threads interlacing in woven fabric (single and multi-woven fabric), is described in various articles in various ways, according to the purpose of expression and application, as well as the author's erudition. Weave - its repeat as input parameters for definition of construction parameters is often used as graphically presentation of pattern. It is a simple and illustrative, but not an operative way for description of threads interlacing that would allow the analysis of the relationship between the interlacing of ends and picks and the properties of the resulting woven fabric. More complex ways for description of woven fabric structure are mathematical methods using analytic expression of shape of binding wave in warp and weft threads of woven fabric.
Here, a description is required from which can be determined geometric parameters such as the length of the threads in the binding wave, the height of the binding waves, the interlacing angles, the deformation of the fabric in the longitudinal and transversal directions, etc. These parameters can be used for evaluation of the behaviour of woven fabric in the steady state and in the state of woven fabric formation on weaving machine. The condition for such using, however, is the greatest analytics' in the mathematical sense of the description, i.e. the continuity of the interlacing curves in the higher derivations. Most of the well-known mathematical descriptions do not create it, although the derivation of the interlacing model description is sometimes highly sophisticated.
Using the Fourier series it provides a description of the threads interlacing in whole repeat of weave in the longitudinal and transversal directions of single and multi-woven fabrics. As a result, it is possible to create the description where the functions are continuous, they can be arranged into the mathematical characteristics of the various types of interlacing. It is also possible to operate in the mechanics for analysing of the behaviour of the fabrics, depending on the interlacing manners.

However, it is not enough to apply the Fourier series for the threads interlacing description at the level of the equations as listed in the literature, the sine and cosine components describing the series. It requires a deeper theoretical, computer and experimental study on specific woven fabrics. What shape the amplitude and phase characteristics have and how they expand based on individual types of interlacing of multi-woven stitched construction, their behaviour in terms of properties and application possibilities.

The volume of the presented work shows the necessary extent of the basic aspects of the study. The work includes the analysis and utilization of Fourier series in description of plain weave in single fabric as well as the description of multi-woven fabrics - double layers stitched woven fabric.

The main focus of the thesis is the development of basic relations for the description of a) the theoretical definition of the geometry of the double layers stitched woven fabrics and the creation of the theoretical model for the description of the threads interlacing by using the Fourier series, b) the fabrication of the fabrics and the determination of the characteristics for different position if stitching places, c) the comparison of theoretical models with the experimentally determined complete binding profiles and their numerically obtained Fourier parameters.

From the results it is evident that the Fourier characteristics converge with a certain regularity with the possibility of their application for definition of structure of double layers stitched woven fabric.

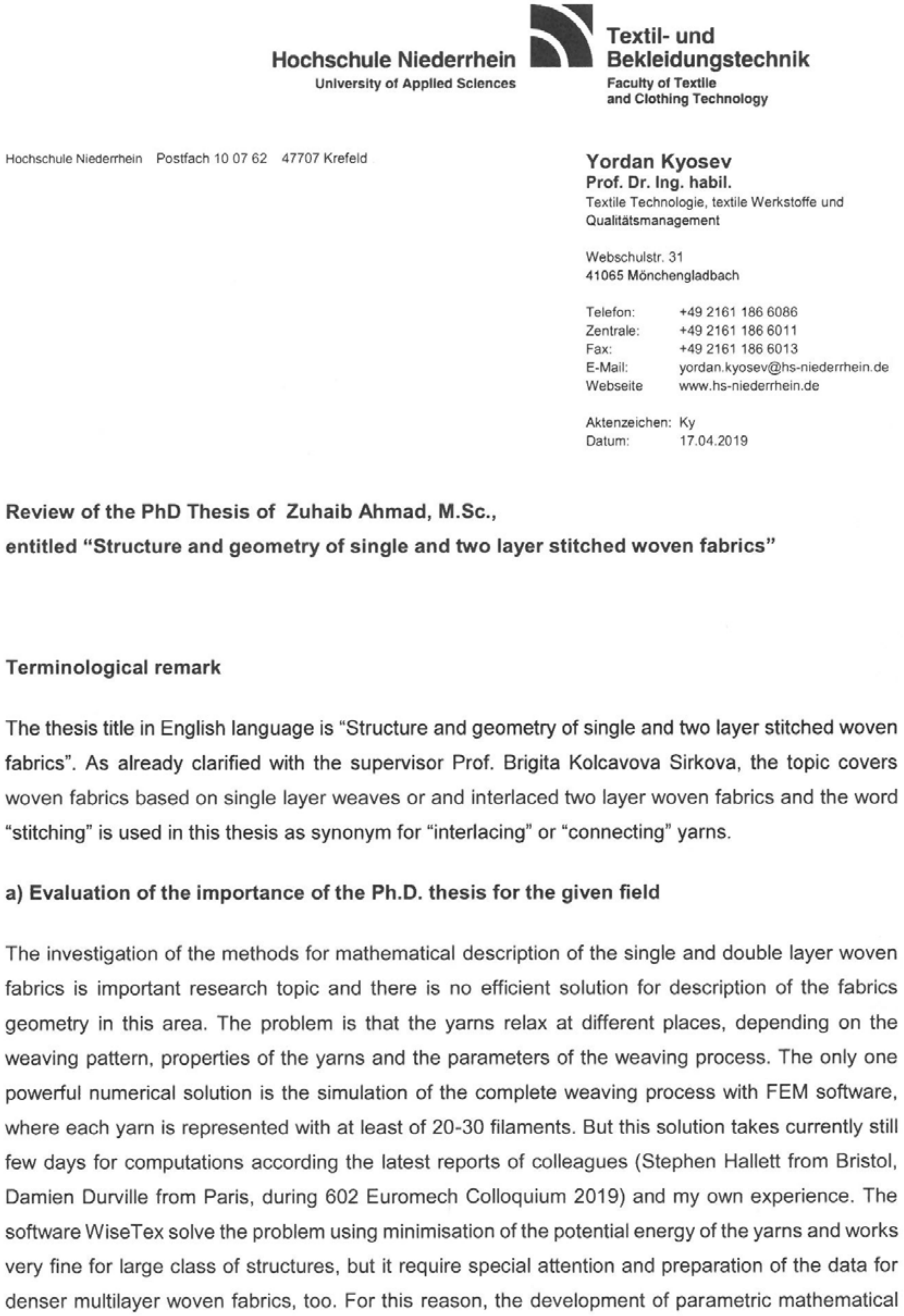
The present work offers a wide application in description of woven fabric structure from a point of view of threads interlacing. In my opinion, the idea of this work can be considered to be beneficial and very useful in the future. The student showed great diligence in the preparation of the work and also achieved good formal levels of the submitted work.

I can recommend the work for the defence and based on a successful defence to give degree Ph.D. to a student Zuhaib AHMAD.

Liberec 18.12.2018

Ing. Brigita KOLČAVOVÁ SIRKOVÁ, Ph.D.

Opponent’s reviews



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Aktenzeichen: Ky
Datum: 17.04.2019

Review of the PhD Thesis of Zuhaib Ahmad, M.Sc., entitled “Structure and geometry of single and two layer stitched woven fabrics”

Terminological remark

The thesis title in English language is “Structure and geometry of single and two layer stitched woven fabrics”. As already clarified with the supervisor Prof. Brigita Kolcavova Sirkova, the topic covers woven fabrics based on single layer weaves or and interlaced two layer woven fabrics and the word “stitching” is used in this thesis as synonym for “interlacing” or “connecting” yarns.

a) Evaluation of the importance of the Ph.D. thesis for the given field

The investigation of the methods for mathematical description of the single and double layer woven fabrics is important research topic and there is no efficient solution for description of the fabrics geometry in this area. The problem is that the yarns relax at different places, depending on the weaving pattern, properties of the yarns and the parameters of the weaving process. The only one powerful numerical solution is the simulation of the complete weaving process with FEM software, where each yarn is represented with at least of 20-30 filaments. But this solution takes currently still few days for computations according the latest reports of colleagues (Stephen Hallett from Bristol, Damien Durville from Paris, during 602 Euromech Colloquium 2019) and my own experience. The software WiseTex solve the problem using minimisation of the potential energy of the yarns and works very fine for large class of structures, but it require special attention and preparation of the data for denser multilayer woven fabrics, too. For this reason, the development of parametric mathematical method and numerical procedure for modelling of the geometry of the multilayer woven structures can speed up the design of such fabrics and optimisation of their properties. This thesis contributes to this area with a different method, which can be used for certain type of applications.

b) Problem-solving procedure, methods and achievement of the stated objective

The author selected Fourier series for approximation of the yarn axis. He recognize the different regions - single layer structure and connecting ("stitched") regions - and analyse these carefully. The identification of the yarn axis was done using modern image processing tool NIS after weaving, preparation, cutting and creating of microscopic images of seven different samples. The coefficients in the Fourier series are obtained using "theoretical FS analysis". How exactly the FFT is performed, remains not clear for me, but I can suppose, that the NIS software provides build in FFT analysis or any other computational tool, available today, was be used.

The selected procedures and methods for the achievement of the objective are not most flexible and the most powerful for this task, but their selection is reasonable and applicant demonstrate the ability to choice suitable modern tools and methods and apply them successfully for investigation of research problems.

c) Opinion on the results of the Ph.D. thesis and the importance of the author's specific contribution

The author demonstrate, that using image processing and analysing with Fourier methods is possible to represent effectively the yarn axis of multilayer woven fabrics. He succeed to apply the method for single and two connected layers and the developed method can be extended for automatic creation of 3D geometries based on the weaving pattern.

d) Other statements concerning mainly the evaluation of the method, clarity of structure, layout and the language level of the Ph.D. thesis

The structure of the thesis is clear. The language can be improved in several places, but is readable and understandable. I would expect to see more careful evaluation of the method for more complex pattern. The current evaluation of the quality of the approximation was based on few periods of the weaves and is mainly subjective (visual) and limited for few types of woven fabrics and materials. Any comparison to more simple approximation techniques like using "just splines" between the contact points is missing, but would be valuable, because it could demonstrate the advantages and problems of the single methods. Ideas for application of the obtained analytical functions are missing too –

these can be applied directly for computation of the crimp / yarn length and probably some more parameters of the structures analytically.

e) Comments on the student's publications

The student has 7 publication of good journals, 4 submitted publication in high impact journals under review and 7 publications in international conferences, all with the supervisor or different co-authors. This is very good publication record for the level of a PhD student.

f) The opponent's unambiguous statement whether he recommends the Ph.D. thesis for defence.

Based on the reading of the thesis and above short remarks, I would recommend to the Dean and the PhD Board to accept the thesis for defence.

Best regards

Prof. Dr.-Ing. habil. Yordan Kyosev

17.4.2019

Thesis title: Structure and geometry of single and two layer stitched woven fabrics

Student: Zuhaib Ahmad, M.Sc.

A. The thesis presents interesting topic and a comprehensive work of theoretical and experimental investigation of woven structures. This is an important field in the understanding of woven structures for applications in industrial materials including composites.

B. The objectives are clear. They are well defined and outlined. The thesis describes systematic methods of theoretical evaluation.
The experimental part is limited. The number of samples developed to validate the theoretical models is relatively small.

C. A variety of theoretical models starting from Pierce, Kemp and Olofsson are stated. Their specific application in special structural geometries are outlined. Extensive application of DFT (Discrete Fourier Transformation) is explored.

D. The method of evaluation is proven and correctly selected. Language is in good level.

E. The candidates has 1 publications in Journal of Textile Institute, 1 in Industria Textila, 1 in Vlakna and Textil and 1 in Material Science and Engg. For defence of PhD thesis this is quite sufficient. It must be outlined that in this area of research, it is relatively difficult to publish large number of articles. Other articles are under review and expected to be published in near future.

F. I would suggest the candidate to address to the following points during the defence.

1. What are the difficulties in modeling woven fabric structures using simple geometry?

2. The importance of computational tools (e.g. FEM, DFT etc.) to solve the structural problems should be highlighted. These tools make the solution of fabric geometry much simpler and user friendly.

3. What assumptions are necessary in modeling the fabric geometries with multifilament Basalt yarns/Tows? Especially with respect to the cross-sectional deformation?

4. What is the error of prediction with regard to the data obtained from real fabric samples? How does the candidate minimize such error in his models?

5. What can be application of composites made from such single layer and 2 layer structures?

The work has fulfilled requirements of PhD thesis and in view of the methodology adopted and obtained results, I recommend it for the Defence.

doc. Rajesh Mishra, PhD
26.03.2019